EE 350 Engineering E & M

Handout 7 – Magnetostatics part 1: basics

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Outline:

Part 1: (quite similar to electrostatics counterpart; question: <u>what are the major difference?</u>)

0. Recap: Electrostatics & Magnetostatics
    a. Electrostatics & Magnetostatics

$$\begin{cases} \left.\begin{array}{l} \nabla\cdot\vec{D}=\rho_v \\ \nabla\times\vec{E}=0 \end{array}\right\} & \text{Electrostatics (chapter 4)} \\ \left.\begin{array}{l} \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{H}=\vec{J} \end{array}\right\} & \text{Magnetostatics (chapter 5)} \end{cases}$$

    b. Integral forms?

    c. Remarks
        i. R1: The static cases happen when all charges are permanently fixed in space, or their movement is at a steady rate (steady current).
        ii. R2: In the static case the E-field and M-field are decoupled.

1. The electromagnetic force:

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q\vec{u}\times\vec{B} = q(\vec{E}+\vec{u}\times\vec{B}) \sim \text{Lorentz force}$$

$\vec{u}$: velocity of q.

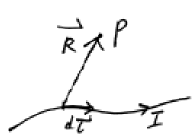
2. Current-carrying conductor experiences magnetic force

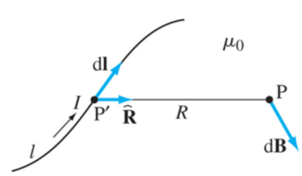
$$\vec{F}_m = I\int_L d\vec{l}\times\vec{B} \qquad (\text{note } d\vec{l} \text{ has direction of current})$$

3. Magnetic field can be generated by a steady current

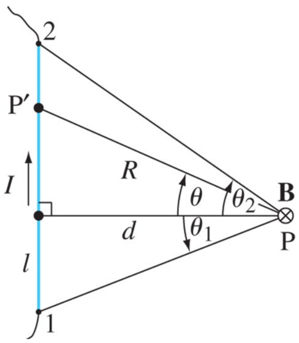
$$d\vec{H} = \frac{I}{4\pi}\frac{d\vec{l}\times\hat{R}}{R^2} \ (A/m) \sim \text{Biot-Savart law}$$

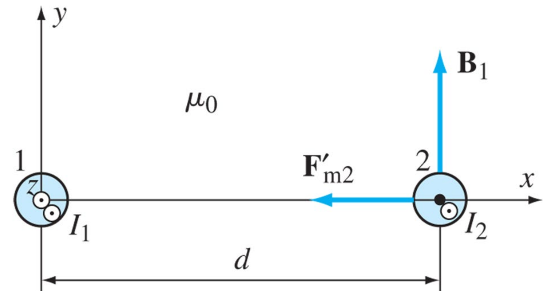
$$\text{or, } \vec{H} = \frac{I}{4\pi}\int_L \frac{d\vec{l}\times\hat{R}}{R^2} \ (A/m)$$





4. Example : two parallel conductor (extension to example 4.2, example 4.6)





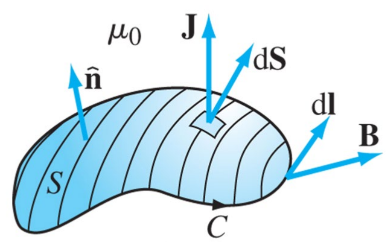
Part 2: Laws in integral forms

0. Recap: The electromagnetic force,
   Current-carrying conductor experiences magnetic force
   Magnetic field can be generated by a steady current

1. Gauss's law for magnetism (no isolated magnetic poles)

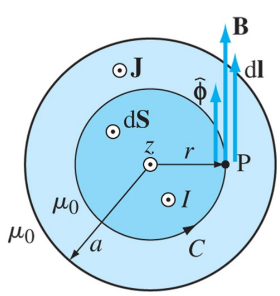
$$\nabla\cdot\vec{B}=0 \iff \oint_S \vec{B}\cdot d\vec{s}=0$$

2. Ampere's law



$$\nabla\times\vec{H}=\vec{J} \iff \oint_L \vec{H}\cdot d\vec{l}=I$$

1). Example: Revisit example 4.6; example 4.9



Part 3: Material constitutive relations; Boundary conditions

0. Recap: The electromagnetic force,
   Current-carrying conductor experiences magnetic force
   Magnetic field can be generated by a steady current
   Gauss's law for magnetism (no isolated magnetic poles)
   Ampere's law

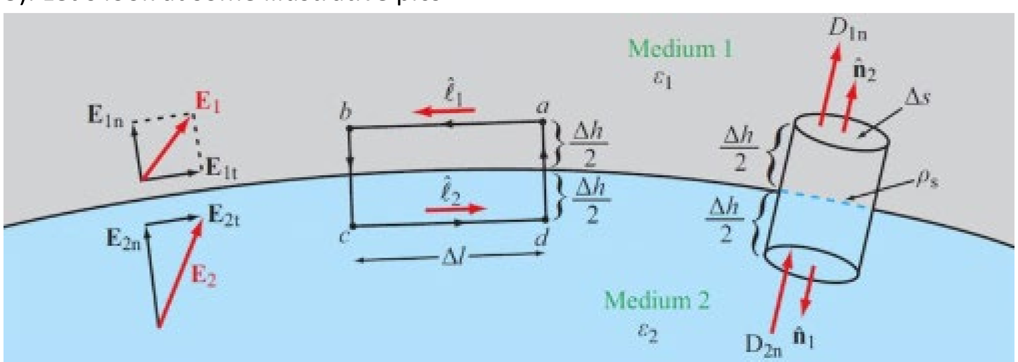
1. Materials macroscopic properties ~ the media constitutive relations
   (We only discuss non Ferromagnetic materials)

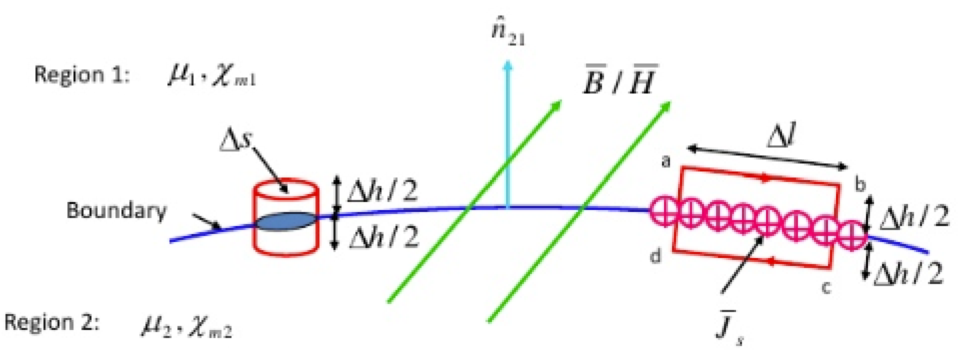
   - For vacuum, $\vec{B} = \mu_0 \vec{H}$, thus $\nabla \cdot \vec{H} = 0$
   - $\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0(1 + \chi_m)\vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$, thus $\nabla \times \vec{B} = \mu \vec{J}$

2. Boundary conditions – can you derive them (from the results of electrostatics)?

By analogy to the e-boundary conditions, we have $B_{1n} - B_{2n} = 0$ and $H_{1t} - H_{2t} = J_l$, with $I = J_l \Delta l$ (Ampere)

Now let's get stronger semantics so you don't need to test the directions of unit vectors:
1). For normal B components across the boundary you don't need to worry, because the result is simply that the B component is continuous across the boundary
2). For tangential components, vector formula is $\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_l$, where $\hat{n}$ is pointing from medium 2 to medium 1 on the boundary.
3). Let's look at some illustrative pics





(note in pic he uses $\vec{J}_s$ which is confusing, because it looks like a surface current density by the notation, but it shall be a line current density).

3. The inductor (not required)

4. Magnetic energy – can you write them out (based on the results from electrostatics) ?

Electrical: $\frac{1}{2}\vec{D}\cdot\vec{E}$ and here: $\frac{1}{2}\vec{B}\cdot\vec{H}$

Part 4: Next we talk about some advanced topics; and HW 2 assignment.