EE 350 Engineering E & M

Handout 10 – Time-varying fields part 2 + HW 3

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Outline:

0. Recap: Electrostatics & Magnetostatics
   a. Electrostatics & Magnetostatics

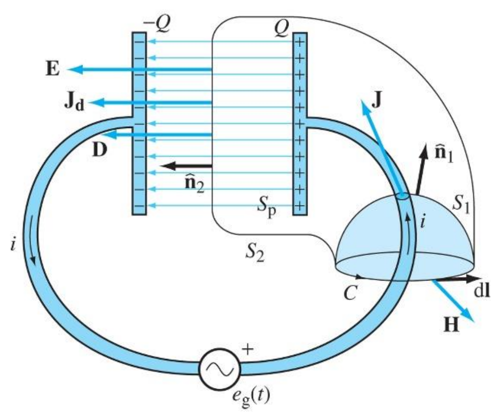
$$\begin{cases} \left.\begin{array}{l} \nabla \cdot \vec{D} = \rho_v \\ \nabla \times \vec{E} = 0 \end{array}\right\} \text{Electrostatics (chapter 4)} \\ \left.\begin{array}{l} \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{H} = \vec{J} \end{array}\right\} \text{Magnetostatics (chapter 5)} \end{cases}$$

   b. Integral forms?

1. The big picture
   a. Maxwell's equations (governing all EM phenomena)

$$\begin{cases} \nabla \cdot \vec{D} = \rho_v \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{H} = \vec{J} + \boxed{\dfrac{\partial \vec{D}}{\partial t}} \end{cases}$$

   b. Remarks
      i. Here we take into account the last term in the 4th eq. in the full set of the Maxwell's equations.



Application of Amp�re's law to a circuit with an air-filled capacitor and time-varying current.

      ii. It is termed as displacement current
      iii. The addition of the term is due to Maxwell and it enables modeling of electromagnetic wave propagation and radiation.
      iv. Note it is Heaviside (also in the backcover) who reduced the number of Maxwell's equations to 4 equations, due to his invention of vector calculus.

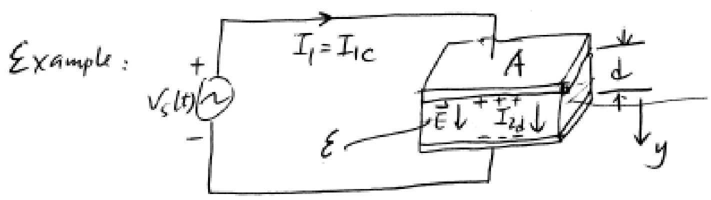
2. Some calculations

1. $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ (Ampère's law) $\Longleftrightarrow \oint_C \vec{H} \cdot d\vec{l} = \underbrace{I_c}_{\text{Conduction current}} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$

Let's call $\int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$ the $\boxed{\text{displacement current}}$, I_D

$\left(I_D \triangleq \int_S \vec{J}_d \cdot d\vec{s} = \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s} \right) \sim \oint_C \vec{H} \cdot d\vec{l} = I_c + I_d$

Example:



$V_s(t) = V_0 \cos \omega t$

① $\Rightarrow I_{1c} = C \frac{dV_c}{dt} = C \frac{dV_s}{dt} = -C V_0 \omega \sin \omega t$

Let's assume the wire is perfect conductor $\Rightarrow \vec{D}_1 = \vec{E}_1 = 0$

so $I_{1d} = 0$, so $I_1 = I_{1c} + I_{1d} = -C V_0 \omega \sin \omega t$

② $I_{2c} = 0$ due to the dielectric material between the 2 plates.

$\vec{E}_2 = \hat{y} \frac{V_c}{d} = \hat{y} \frac{V_0}{d} \cos \omega t$

So $I_{2d} = \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s} = \int \underbrace{\frac{\partial}{\partial t} \left[\hat{y} \frac{\varepsilon V_0}{d} \cos \omega t \right]}_{\hat{y} \frac{\varepsilon V_0}{d} (-\omega \sin \omega t)} \cdot \hat{y} \, ds$

$= -\frac{\varepsilon A}{d} V_0 \omega \sin \omega t = -C V_0 \omega \sin \omega t$

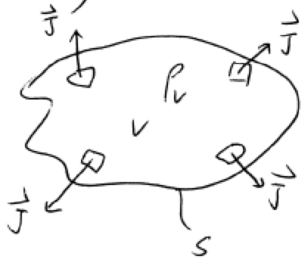
Remarks:

i. Even though the displacement current does not carry real free charge, it nonetheless behaves like a real current.

ii. $\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t} = \vec{J}_{d0} + \vec{J}_p$

3. K's current law:

Charge-Current Continuity relation



$$I = -\frac{dQ}{dt} = -\frac{d}{dt}\int_v \rho_v \, dv$$

$$\Rightarrow \oint_s \vec{J}\cdot d\vec{s} = -\frac{d}{dt}\int \rho_v \, dv = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

$$\Rightarrow \int_v \nabla\cdot\vec{J} \, dv = -\int \frac{\partial \rho_v}{\partial t} dv$$

$$\Rightarrow \boxed{\nabla\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t}}$$

if $\frac{\partial \rho_v}{\partial t} = 0$, and if $\vec{J}_D = 0$ (perfect conductor)

then $\nabla\cdot\vec{J}_c = 0 \sim \underbrace{\oint_s \vec{J}_c\cdot d\vec{s} = 0}_{\text{or } \sum_i I_i = 0} \sim$ (Kirchoff's current law

Homework 3 assignment: due on Monday March 2 before class.

1. Example 6.8: note that in class we used Lenz's law to decide the polarity or the sign of the generated (induced) voltage. I here add a question about how do you decide the polarity or the sign with Lenz's law, specifically for this problem. Draw your $i(t)$ and $v(t)$ vs. t curves to assist your explanation – as what we did in class.
2. Example 6.9
3. Example 6.11

4. Example 7.17 : note we did not discuss mutual inductance, so there is additional knowledge here for which self-study will be needed.

5. Example 8.1
6. Example 8.2
7. Example 8.4
8. Problem 8.1
9. Problem 8.2

10. Read the book or other reference materials, study and summarize on electrical scalar potential and magnetic vector potential. This problem accounts for 20 points. Format will account for additional 5 points: MS word document with professional equation typing is required.