```
(*  #2. Example given in class.  *)


(Clear[A]; Clear[B]; Label[1]; A = Input[{a, b, c}];
  If[Or[A[[1]] ≤ 0, A[[2]]^2 - 4 * A[[1]] * A[[3]] ≥ 0],
   Print["bad input ", A]; Clear[A]; Goto[1]]);
(Label[2]; Print[A];


 k1 = Ceiling[(A[[1]] - A[[2]]) / (2 A[[1]])] - 1;
 k2 = Ceiling[(A[[1]] + A[[2]]) / (2 A[[1]])] - 1;

 If[Or[A[[1]] > A[[3]], And[A[[1]] == A[[3]], A[[1]] ≥ -A[[2]], A[[2]] < 0]],
  B = {A[[3]], -A[[2]], A[[1]]}; A = B; Goto[2]];

 If[Or[And[A[[1]] == A[[3]], A[[2]] < -A[[1]]], And[A[[1]] < A[[3]], A[[2]] < -A[[1]]]],
  B = {A[[1]], A[[2]] + 2 k1 * A[[1]], A[[1]] k1^2 + A[[2]] k1 + A[[3]]}; A = B; Goto[2]];

 If[Or[And[A[[1]] == A[[3]], A[[2]] > A[[1]]], And[A[[1]] < A[[3]], A[[2]] > A[[1]]]],
  B = {A[[1]], A[[2]] - 2 k2 * A[[1]], A[[1]] k2^2 - A[[2]] k2 + A[[3]]}; A = B; Goto[2]];

 If[And[A[[1]] < A[[3]], A[[2]] == -A[[1]]],
  B = {A[[1]], A[[1]], A[[3]]}; A = B; Goto[2]];



 Print["Reduced"])
```

```
{37, 59, 25}

{25, -59, 37}

{25, -9, 3}

{3, 9, 25}

{3, 3, 19}

Reduced
```

(* #3. I added an output of the running product of the U's at each step. The U we seek is then the last U outputted. See below.

To find the minimum positve f(x,y), we could use the inequalities $y^2 < \frac{4aM}{-d}$ and $(2ax+by)^2 < aM+dy^2$ to determine the x and y that produce it. But, since we already know U and the fact that the minimum positive f is "a" when f is reduced, we can read off our output and see that the minimum positive f is 21 and then we can simply multiply U.(1,0) to get (x,y), which is (-3,4). Of course, (3,-4) would also work. *)

```
(Clear[A]; Clear[B]; Label[1]; A = Input[{a, b, c}];
  If[Or[A[[1]] ≤ 0, A[[2]]^2 - 4 * A[[1]] * A[[3]] ≥ 0],
   Print["bad input ", A]; Clear[A]; Goto[1]]);
(U = {{1, 0}, {0, 1}}; Label[2]; Print[A, "   U=", U];

 k1 = Ceiling[(A[[1]] - A[[2]]) / (2 A[[1]])] - 1;
 k2 = Ceiling[(A[[1]] + A[[2]]) / (2 A[[1]])] - 1;

 If[Or[A[[1]] > A[[3]], And[A[[1]] == A[[3]], A[[1]] ≥ -A[[2]], A[[2]] < 0]],
  B = {A[[3]], -A[[2]], A[[1]]}; A = B; U = U.{{0, 1}, {-1, 0}}; Goto[2]];

 If[Or[And[A[[1]] == A[[3]], A[[2]] < -A[[1]]], And[A[[1]] < A[[3]], A[[2]] < -A[[1]]]],
  B = {A[[1]], A[[2]] + 2 k1 * A[[1]], A[[1]] k1^2 + A[[2]] k1 + A[[3]]};
  A = B; U = U.{{1, k1}, {0, 1}}; Goto[2]];

 If[Or[And[A[[1]] == A[[3]], A[[2]] > A[[1]]], And[A[[1]] < A[[3]], A[[2]] > A[[1]]]],
  B = {A[[1]], A[[2]] - 2 k2 * A[[1]], A[[1]] k2^2 - A[[2]] k2 + A[[3]]};
  A = B; U = U.{{1, -k2}, {0, 1}}; Goto[2]];

 If[And[A[[1]] < A[[3]], A[[2]] == -A[[1]]],
  B = {A[[1]], A[[1]], A[[3]]}; A = B; U = U.{{1, 1}, {0, 1}}; Goto[2]];

 Print["Reduced"])
{12 345, 18 309, 6789}"   U="{{1, 0}, {0, 1}}
{6789, -18 309, 12 345}"   U="{{0, 1}, {-1, 0}}
{6789, -4731, 825}"   U="{{0, 1}, {-1, -1}}
{825, 4731, 6789}"   U="{{-1, 0}, {1, -1}}
{825, -219, 21}"   U="{{-1, 3}, {1, -4}}
{21, 219, 825}"   U="{{-3, -1}, {4, 1}}
{21, 9, 255}"   U="{{-3, 14}, {4, -19}}
"Reduced"

(*  Check  *)

X = {{-3, 14}, {4, -19}}.{x, y}
{-3 x + 14 y, 4 x - 19 y}
f[X_] := 12 345 X[[1]]^2 + 18 309 * X[[1]] * X[[2]] + 6789 * X[[2]]^2
ExpandAll[f[X]]
21 x^2 + 9 x y + 255 y^2


(* #4.  Outputted a few primes. Enter 0 to stop program. *)


Clear[A, p, a, c1, c2, c3, c4, B1, B2, B3, B4]; (Label[0];
 Clear[A]; Clear[p]; p = Input[p];
 If[p == 0, Goto[7]];
 If[And[PrimeQ[p], p != 2], Goto[1], Print["bad input"]; Goto[0]];

 Label[1];
 Do[If[Mod[x^2 + y^2 + 1, p] == 0, A[1] = x; A[2] = 1; A[3] = y; A[4] = 0; Goto[3]],
  {x, 0, (p - 1) / 2}, {y, 0, (p - 1) / 2}];

 Label[3];
 m = (A[1]^2 + A[2]^2 + A[3]^2 + A[4]^2) / p;


 Label[4];

 If[m == 1, Goto[6]];
 If[OddQ[m], Goto[5]];
 Ev = {}; Od = {};
 Do[If[EvenQ[A[i]], Ev = Append[Ev, A[i]], Od = Append[Od, A[i]]], {i, 1, 4}];

 If[Or[Length[Ev] == 4, Length[Ev] == 0], B1 = (A[1] + A[2]) / 2; B2 = (A[1] - A[2]) / 2;
  B3 = (A[3] + A[4]) / 2; B4 = (A[3] - A[4]) / 2, B1 = (Ev[[1]] + Ev[[2]]) / 2;
  B2 = (Ev[[1]] - Ev[[2]]) / 2; B3 = (Od[[1]] + Od[[2]]) / 2; B4 = (Od[[1]] - Od[[2]]) / 2];

 m = m / 2; A[1] = B1; A[2] = B2; A[3] = B3; A[4] = B4;
 Goto[4];



 Label[5];

 Do[a[i] = Mod[A[i], m, -(m - 1) / 2], {i, 1, 4}];
 n = a[1]^2 + a[2]^2 + a[3]^2 + a[4]^2;

 k = n / m;

 c1 = (a[1] A[1] + a[2] A[2] + a[3] A[3] + a[4] A[4]) / m;
 c2 = (a[1] A[2] - a[2] A[1] + a[4] A[3] - a[3] A[4]) / m;
 c3 = (a[1] A[3] - a[3] A[1] + a[2] A[4] - a[4] A[2]) / m;
 c4 = (a[1] A[4] - a[4] A[1] + a[3] A[2] - a[2] A[3]) / m;

 A[1] = c1; A[2] = c2; A[3] = c3; A[4] = c4; m = k;
 Goto[4];

 Label[6];

 Print["p=", p, "~{", A[1], ",", A[2], ",", A[3], ",", A[4], "}"];
 Goto[0];
 Label[7];
)
```

```
"p="5"~{"0","1","2","0"}"
"p="17"~{"0","1","4","0"}"
"p="101"~{"0","1","10","0"}"
```

```
"p="997"~{"31","6","0","0"}"
"p="7919"~{"42","-37","-5","69"}"
"p="10 103"~{"63","-63","-41","22"}"
"p="17 389"~{"-125","42","0","0"}"
"p="27 449"~{"0","0","160","-43"}"

(*  Check  *)

31^2 + 6^2
997
63^2 + 63^2 + 41^2 + 22^2
10 103
42^2 + 37^2 + 5^2 + 69^2
7919
125^2 + 42^2
17 389
160^2 + 43^2
27 449


(*  #5.  Program with various inputs. Enter 0 to stop
 program. Note: Exponents must be >0, therefore will not accept 1 as an input. *)


(Label[10]; Clear[n0, A, a]; n0 = Input[N]; If[n0 == 0, Goto[100]];
  m0 = Length[n0];
  Do[If[Or[Not[PrimeQ[n0[[j, 1]]]], n0[[j, 2]] < 1, Not[IntegerQ[n0[[j, 2]]]]],
    Print["bad input"]; Goto[10]], {j, 1, m0}];

  Do[If[n0[[j, 1]] == 2, A[j, 1] = 1; A[j, 2] = 1; A[j, 3] = 0; A[j, 4] = 0,

     (p = n0[[j, 1]];
      Label[1];
      Do[If[Mod[x^2 + y^2 + 1, p] == 0, A[j, 1] = x; A[j, 2] = 1;
         A[j, 3] = y; A[j, 4] = 0; Goto[3]], {x, 0, (p - 1) / 2}, {y, 0, (p - 1) / 2}];

      Label[3];
      m = (A[j, 1]^2 + A[j, 2]^2 + A[j, 3]^2 + A[j, 4]^2) / p;


      Label[4];

      If[m == 1, Goto[6]];
      If[OddQ[m], Goto[5]];
      Ev = {}; Od = {};
      Do[If[EvenQ[A[j, i]],
         Ev = Append[Ev, A[j, i]], Od = Append[Od, A[j, i]]], {i, 1, 4}];

      If[Or[Length[Ev] == 4, Length[Ev] == 0], B1 = (A[j, 1] + A[j, 2]) / 2;
       B2 = (A[j, 1] - A[j, 2]) / 2; B3 = (A[j, 3] + A[j, 4]) / 2; B4 = (A[j, 3] - A[j, 4]) / 2,
    B1 = (Ev[[1]] + Ev[[2]]) / 2; B2 = (Ev[[1]] - Ev[[2]]) / 2;
    B3 = (Od[[1]] + Od[[2]]) / 2; B4 = (Od[[1]] - Od[[2]]) / 2];

   m = m / 2; A[j, 1] = B1; A[j, 2] = B2; A[j, 3] = B3; A[j, 4] = B4;
   Goto[4];



   Label[5];

   Do[a[j, i] = Mod[A[j, i], m, -(m - 1) / 2], {i, 1, 4}];
   n = a[j, 1]^2 + a[j, 2]^2 + a[j, 3]^2 + a[j, 4]^2;

   k = n / m;

   c1 = (a[j, 1] A[j, 1] + a[j, 2] A[j, 2] + a[j, 3] A[j, 3] + a[j, 4] A[j, 4]) / m;
   c2 = (a[j, 1] A[j, 2] - a[j, 2] A[j, 1] + a[j, 4] A[j, 3] - a[j, 3] A[j, 4]) / m;
   c3 = (a[j, 1] A[j, 3] - a[j, 3] A[j, 1] + a[j, 2] A[j, 4] - a[j, 4] A[j, 2]) / m;
   c4 = (a[j, 1] A[j, 4] - a[j, 4] A[j, 1] + a[j, 3] A[j, 2] - a[j, 2] A[j, 3]) / m;

   A[j, 1] = c1; A[j, 2] = c2; A[j, 3] = c3; A[j, 4] = c4; m = k;
   Goto[4];

   Label[6];
  )], {j, 1, m0}];


Do[
 c1 = 1; c2 = 0; c3 = 0; c4 = 0;

 Do[
  d1 = (c1 * A[j, 1] + c2 * A[j, 2] + c3 * A[j, 3] + c4 * A[j, 4]);
  d2 = (c1 * A[j, 2] - c2 * A[j, 1] + c4 * A[j, 3] - c3 * A[j, 4]);
  d3 = (c1 * A[j, 3] - c3 * A[j, 1] + c2 * A[j, 4] - c4 * A[j, 2]);
  d4 = (c1 * A[j, 4] - c4 * A[j, 1] + c3 * A[j, 2] - c2 * A[j, 3]);

  c1 = d1; c2 = d2; c3 = d3; c4 = d4,
  {r, 1, n0[[j, 2]]}];
 A[j, 1] = c1; A[j, 2] = c2; A[j, 3] = c3; A[j, 4] = c4,
 {j, 1, m0}];


c1 = 1; c2 = 0; c3 = 0; c4 = 0;

Do[
 d1 = (c1 * A[j, 1] + c2 * A[j, 2] + c3 * A[j, 3] + c4 * A[j, 4]);
 d2 = (c1 * A[j, 2] - c2 * A[j, 1] + c4 * A[j, 3] - c3 * A[j, 4]);
 d3 = (c1 * A[j, 3] - c3 * A[j, 1] + c2 * A[j, 4] - c4 * A[j, 2]);
 d4 = (c1 * A[j, 4] - c4 * A[j, 1] + c3 * A[j, 2] - c2 * A[j, 3]);
```

```
      c1 = d1; c2 = d2; c3 = d3; c4 = d4,


      {j, 1, m0}];
    M = 1;
    Do[M = M * n0[[j, 1]]^n0[[j, 2]], {j, 1, m0}];
    Print["N=", M, "=", n0, "~{", c1, ",", c2, ",", c3, ",", c4, "}"]; Goto[10];
    Label[100]);
```

```
"N="720"="{{2, 4}, {3, 2}, {5, 1}}"~{"0","12","24","0"}"
"N="2"="{{2, 1}}"~{"1","1","0","0"}"
"N="1693185147"="{{3, 3}, {7919, 2}}"~{"23757","-23757","-23757","0"}"
"N="128"="{{2, 7}}"~{"8","8","0","0"}"
```

```
(*  Check  *)
12^2 + 24^2
720
23757^2 + 23757^2 + 23757^2
1693185147
8^2 + 8^2
128
```

```
(*  #6.  Changed program so we just need to enter a nonnegative
 integer (i.e. just enter the integer, not its UPF). Output
 shown here is for input 123456789. Enter 0 to stop program. *)
```

```
(Label[10]; Clear[n0, A, a]; n0 = FactorInteger[Input[N]]; If[n0 == {{0, 1}}, Goto[100]];
  If[n0 == {}, Print["N=", 1, "~{", 1, ",", 0, ",", 0, ",", 0, "}"]; Goto[10]];
  m0 = Length[n0];
  Do[If[Or[Not[PrimeQ[n0[[j, 1]]]], n0[[j, 2]] < 1, Not[IntegerQ[n0[[j, 2]]]]],
     Print["bad input"]; Goto[10]], {j, 1, m0}];

  Do[If[n0[[j, 1]] == 2, A[j, 1] = 1; A[j, 2] = 1; A[j, 3] = 0; A[j, 4] = 0,

      (p = n0[[j, 1]];
       Label[1];
       Do[If[Mod[x^2 + y^2 + 1, p] == 0, A[j, 1] = x; A[j, 2] = 1;
          A[j, 3] = y; A[j, 4] = 0; Goto[3]], {x, 0, (p - 1) / 2}, {y, 0, (p - 1) / 2}];

       Label[3];
       m = (A[j, 1]^2 + A[j, 2]^2 + A[j, 3]^2 + A[j, 4]^2) / p;


       Label[4];

       If[m == 1, Goto[6]];
       If[OddQ[m], Goto[5]];
```

```
   Ev = {}; Od = {};
   Do[If[EvenQ[A[j, i]],
     Ev = Append[Ev, A[j, i]], Od = Append[Od, A[j, i]]], {i, 1, 4}];

   If[Or[Length[Ev] == 4, Length[Ev] == 0], B1 = (A[j, 1] + A[j, 2]) / 2;
    B2 = (A[j, 1] - A[j, 2]) / 2; B3 = (A[j, 3] + A[j, 4]) / 2; B4 = (A[j, 3] - A[j, 4]) / 2,
    B1 = (Ev[[1]] + Ev[[2]]) / 2; B2 = (Ev[[1]] - Ev[[2]]) / 2;
    B3 = (Od[[1]] + Od[[2]]) / 2; B4 = (Od[[1]] - Od[[2]]) / 2];

   m = m / 2; A[j, 1] = B1; A[j, 2] = B2; A[j, 3] = B3; A[j, 4] = B4;
   Goto[4];



   Label[5];

   Do[a[j, i] = Mod[A[j, i], m, - (m - 1) / 2], {i, 1, 4}];
   n = a[j, 1]^2 + a[j, 2]^2 + a[j, 3]^2 + a[j, 4]^2;

   k = n / m;

   c1 = (a[j, 1] A[j, 1] + a[j, 2] A[j, 2] + a[j, 3] A[j, 3] + a[j, 4] A[j, 4]) / m;
   c2 = (a[j, 1] A[j, 2] - a[j, 2] A[j, 1] + a[j, 4] A[j, 3] - a[j, 3] A[j, 4]) / m;
   c3 = (a[j, 1] A[j, 3] - a[j, 3] A[j, 1] + a[j, 2] A[j, 4] - a[j, 4] A[j, 2]) / m;
   c4 = (a[j, 1] A[j, 4] - a[j, 4] A[j, 1] + a[j, 3] A[j, 2] - a[j, 2] A[j, 3]) / m;

   A[j, 1] = c1; A[j, 2] = c2; A[j, 3] = c3; A[j, 4] = c4; m = k;
   Goto[4];

   Label[6];
  )], {j, 1, m0}];


Do[
 c1 = 1; c2 = 0; c3 = 0; c4 = 0;

 Do[
  d1 = (c1 * A[j, 1] + c2 * A[j, 2] + c3 * A[j, 3] + c4 * A[j, 4]);
  d2 = (c1 * A[j, 2] - c2 * A[j, 1] + c4 * A[j, 3] - c3 * A[j, 4]);
  d3 = (c1 * A[j, 3] - c3 * A[j, 1] + c2 * A[j, 4] - c4 * A[j, 2]);
  d4 = (c1 * A[j, 4] - c4 * A[j, 1] + c3 * A[j, 2] - c2 * A[j, 3]);

  c1 = d1; c2 = d2; c3 = d3; c4 = d4,
  {r, 1, n0[[j, 2]]}];
 A[j, 1] = c1; A[j, 2] = c2; A[j, 3] = c3; A[j, 4] = c4,
 {j, 1, m0}];


c1 = 1; c2 = 0; c3 = 0; c4 = 0;
```

```
    Do[
     d1 = (c1 * A[j, 1] + c2 * A[j, 2] + c3 * A[j, 3] + c4 * A[j, 4]);
     d2 = (c1 * A[j, 2] - c2 * A[j, 1] + c4 * A[j, 3] - c3 * A[j, 4]);
     d3 = (c1 * A[j, 3] - c3 * A[j, 1] + c2 * A[j, 4] - c4 * A[j, 2]);
     d4 = (c1 * A[j, 4] - c4 * A[j, 1] + c3 * A[j, 2] - c2 * A[j, 3]);

     c1 = d1; c2 = d2; c3 = d3; c4 = d4,


     {j, 1, m0}];
    M = 1;
    Do[M = M * n0[[j, 1]]^n0[[j, 2]], {j, 1, m0}];
    Print["N=", M, "~{", c1, ",", c2, ",", c3, ",", c4, "}"]; Goto[10];
    Label[100]);

"N="123 456 789"~{"2223","6324","-8550","-2328"}"


(*  Check  *)
2223^2 + 6324^2 + 8550^2 + 2328^2
123 456 789


(* #7.  Outputted a few primes. Enter 0 to stop program. *)


Clear[A, p, a, c1, c2, c3, c4, B1, B2, B3, B4]; (Label[0];
 Clear[A]; Clear[p]; p = Input[p];
 If[p ⩵ 0, Goto[7]];
 If[And[PrimeQ[p], Mod[p, 4] ⩵ 1], Goto[1], Print["bad input"]; Goto[0]];

 Label[1];
 Do[If[Mod[x^2 + 1, p] == 0, A[1] = x; A[2] = 1; Goto[3]], {x, 0, (p - 1)}];

 Label[3];
 m = (A[1]^2 + A[2]^2) / p;


 Label[4];

 If[m ⩵ 1, Goto[6]];
 If[OddQ[m], Goto[5]];

 B1 = (A[1] + A[2]) / 2; B2 = (A[1] - A[2]) / 2;

 m = m / 2; A[1] = B1; A[2] = B2;
 Goto[4];

 Label[5];
 Do[a[i] = Mod[A[i], m, -(m - 1) / 2], {i, 1, 2}];
 n = a[1]^2 + a[2]^2;

 k = n / m;

 c1 = (a[1] A[1] + a[2] A[2]) / m;
 c2 = (a[1] A[2] - a[2] A[1]) / m;

 A[1] = c1; A[2] = c2; m = k;
 Goto[4];

 Label[6];

 Print["p=", p, "~{", A[1], ",", A[2], "}"];
 Goto[0];
 Label[7];
)
```

```
"p="5"~{"2","1"}"
"p="17"~{"4","1"}"
"p="41"~{"5","4"}"
"p="197"~{"14","1"}"
"p="6997"~{"74","39"}"

(*  Check  *)

14^2 + 1^2
197
74^2 + 39^2
6997
```