Problems to think about solution (Chapter 4)

1. Prove that if p is a prime number that satisfies $p \equiv 1 \bmod 4$ then whenever g is primitive mod p, so is $-g$.

2. Prove that if p and $2p+1$ are prime numbers with $p \equiv 3 \bmod 4$ then -2 is primitive mod $2p+1$.

3. Prove that if p and $p' = 2^k p + 1$ are odd primes then a is a primitive root mod p' if and only if $a^{2^k} \not\equiv 1 \bmod p'$ and $\left(\frac{a}{p'}\right) = -1$.

4. Identify all Fermat and Mersenne primes p that have 2 as a primitive root.