Problem to think about (Chapter 3), revised

1. Prove that if p is a prime number satisfying $p \equiv 3 \bmod 4$, then whenever g is primitive mod p, $-g$ is not primitive mod p.

2. Let p be a prime number and let $(a, p) = 1$. Prove that a is a primitive root mod p if and only if $a^{\frac{p-1}{q}} \not\equiv 1 \bmod p$ for every prime divisor q of $p - 1$.

3. Let p be a prime number and let $a > 1$ be a number that satisfies $a \equiv 1 \bmod p$. Let $o_n(a)$ denote the order of a mod p^n, i.e. the least positive integer k such that $a^k \equiv 1 \bmod p^n$. Let j be the largest positive integer such that $p^j | (a - 1)$. Find a formula for $o_n(a)$ in terms of n and j. A proof is required, not a conjecture, but it helps to work out some examples and make a conjecture first.