

Problems to think about (Chapter 2)

1. Let f and g be multiplicative arithmetic functions. Define an arithmetic function h via

$$h(n) = \sum_{d|P(n)} f(d)g(n/d),$$

where $P(n) = q_1q_2 \cdots q_r$ is the product of the distinct primes dividing n and $P(1) = 1$. Prove that h is a multiplicative function, then find $F_h(t)$ in terms of f and $F_g(t)$.

2. Find a multiplicative function f such that

$$\sum_{d|P(n)} f(d)\sigma(n/d) = 1$$

for all n , where $P(n)$ is as above, and express f in terms of the multiplicative functions we have already studied in the course.

3. Define $f : \mathbb{Z}^+ \rightarrow \mathbb{R}$ by $f(n) = \chi(n \text{ is odd})$. Prove that f is multiplicative and prove that

$$\sum_{d|n} f(d)\phi(n/d) = \begin{cases} n & n \text{ is odd} \\ \frac{n}{2} & n \text{ is even} \end{cases}$$