Exam 3 Math 321 Spring 2007 Name:

Show all work.

1. Let

$$y = \sum_{n=0}^{\infty} c_n x^n$$

be the power series solution to the initial value problem

$$y'' - xy = 0, \qquad y(0) = 1, y'(0) = 1.$$

Compute c_0, c_1, c_2, c_3, c_4, and c_5.

2. Consider the differential equation

$$(H) \qquad x^2y'' + x^2y' + 20y = 0$$

(a) Explain why $x = 0$ is a regular singular point of (H)

(b) Find the two values of r such that $y = x^r \sum_{n=0}^{\infty} c_n x^n$ is a solution to (H). You need only solve the indicial equation, not find the coefficients c_n .

3. Find the general solution to the following system of linear equations using the operator method. Your final answer should have only two arbitrary constants in it.

$$x' - 3y = -6e^t$$
$$y' + 3x = 2e^t$$

4. Use the matrix method (guess $\begin{bmatrix} x \\ y \end{bmatrix} = e^{mt} \begin{bmatrix} P \\ Q \end{bmatrix}$ etc) to find the general solution to system of linear equations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 6 & -0.5 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$