Exam 2 Math 321 Spring 2007 Name:

Show all work.

$$mx'' + ax' + kx = F(t)$$

$$v_1(x) = -\int^x \frac{F(t)y_2(t)\ dt}{a_0(t)W[y_1(t), y_2(t)]}, \qquad v_2(x) = \int^x \frac{F(t)y_1(t)\ dt}{a_0(t)W[y_1(t), y_2(t)]}$$

$$v = \int \frac{\exp\left[-\int \frac{a_1(x)}{a_0(x)}\ dx\right]}{[f(x)]^2}\ dx$$

1. Find a particular solution to the differential equation

$$y'' - y = e^x + \cos x.$$

2. Find the general solution to the Cauchy-Euler equation

$$x^2y'' - xy' + y = 0.$$

3. Given that $y = x + 1$ is a solution of

$$(x+1)^2y'' - 3(x+1)y' + 3y = 0,$$

find a linearly independent solution by reducing the order.

4. A 16 pound weight is attached to the lower end of a coil spring suspended from a fixed support. The weight comes to rest in its equilibrium position, thereby stretching the spring 6 inches The weight is then pulled down 3 inches below its equilibrium position and released at $t = 0$. The medium offers a resistance in pounds numerically equal to $10x'$, where x' is instantaneous velocity in feet per second. Find the position and the velocity of the spring at time $t = 1$.