Math 321 Final Exam Name:

May 9, 2007

Show all work.

1. *Exact differential equation:* Consider the differential equation

$$\frac{e^y}{x}\,dx + (e^y \ln x - 5\sin y)\,dy = 0, \qquad y(1) = 0.$$

(a) Verify that the differential equation is exact.

(b) Solve the differential equation.

2. *Mechanics:* A 320 pound object is dropped with initial velocity of 0 feet per second from a height of 3200 feet. As it falls, air resistance acts upon it, and this resistance (in units of pounds) is numerically equal to one-fifth of velocity (in units of feet per second). Find the velocity of the falling object as a function of t.

3. *Method of Frobenius:* Consider the differential equation

$$xy'' - 2y' + xy = 0.$$

(a) Explain why $x = 0$ is a regular singular point of the differential equation.

(b) Find the two values of r such that $y = x^r \sum_{n=0}^{\infty} c_n x^n$ is a solution to the differential equation.

4. *System of differential equations:* Solve the following system of differential equations, using your method of choice (differential operators, matrix algebra, or Laplace transform). Assume x and y are functions of t.

$$\begin{aligned} x' &= 2x + 7y \\ y' &= 3x + 6y \\ x(0) &= 4 \\ y(0) &= -6 \end{aligned}$$

5. *Laplace transform:* Consider the differential equation

$$y' - y = \begin{cases} 0, & 0 < t < 5, \\ 1, & t > 5, \end{cases} \qquad y(0) = 0.$$

(a) Find $F(s) = \mathcal{L}\{y\}$

(b) Find $y = \mathcal{L}^{-1}\{F(s)\}$