Week 1 Math 247 Sections 1.1, 1.2, 1.3

System of Linear Equations: see Example 1, page 7.

Solution Set: all values of the variables which make the equations true.

Elementary Transformations: operations which convert one system to another without changing the solution set. See page 7.

Illustrate these transformations on Example 1, page 7.

Coefficient Matrix: matrix of coefficients!

Augmented Matrix: Last column includes constants on the right hand side of the equations.

Redo Example 1, page 7, in terms of augmented matrix.

An augmented matrix can always be transformed to reduced echelon form.

Reduced echelon form: defined on page 17.

Leading terms in reduced echelon matrix: the first non-zero coefficient in each row.

Leading variables: the variables corresponding to the leading terms in the reduced echelon matrix.

Method for transforming augmented matrix to reduced echelon form:

1. If necessary, swap rows so that there is a non-zero entry in row 1, column 1.

2. If necessary, divide row 1 by entry in position $(1, 1)$.

3. For each $k > 1$, add multiple of row 1 to row k to create a zero in position row k, column 1.

4. Discard (ignore) row 1 and column 1. Continue operating on remaining submatrix until no longer possible.

5. Note: whenever step 1 is not possible, discard (ignore) this column and move on to the next one.

6. A method which avoids fractions: don't bother to try to get a 1 in the leading positions.

Detecting an infinite number of solutions to a linear system of equations: There will be fewer leading terms than variables.

Characterizing an infinite solution set: Suppose variables x_1 through x_k are leading. Assign arbitrary values $x_{k+1} = r$, $x_{k+2} = s$, $x_{k+3} = t$, and so on, then compute x_1 through x_k using these values.

Example of a linear system with an infinite solution set: example at top of page 24.

Any system of linear equations with fewer leading terms than columns will have an infinite number of solutions. If a system of linear equations has the same number of leading terms as columns, then either there is no solution or exactly one solution.

Curve Fitting: Find the coefficients of a polynomial satisfying $p(x_1) = y_1$ through $p(x_n) = y_n$. Treat this as a system of n equations. To avoid an over-determined system, use a polynomial with n or fewer coefficients. See Example 1, page 29.

Traffic Flow: Flow in must equal flow out at intersections. Use this property to write down flow equations. If there are an infinite number of solutions, use Gaussian Elimination to maximize or minimize one of the flow variables. See the example on page 34.

Homework problems:

Section 1.1, problems 5cf, 6fg, 7cd, 10a, 11ad, 12ae, 13cd

Section 1.2, problems 1cd, 2gh, 3cdf, 6, 8, 10, 13, 14

Section 1.3, problems 5, 6, 16, 18, 21