In[11]:= **InnerProduct = FunctionProduct**

Out[11]= FunctionProduct

In[12]:= **ScalarProduct = FunctionScalarProduct**

Out[12]= FunctionScalarProduct

In[13]:= **VectorSum = FunctionSum**

Out[13]= FunctionSum

In[14]:= **constantRule[x_] := 1**

In[15]:= **constantDomain = {0, 2 Pi}**

Out[15]= {0, 2 π}

In[16]:= **constantFunction = {constantRule, constantDomain}**

Out[16]= {constantRule, {0, 2 π}}

In[17]:= **linearRule[x_] := x**

In[18]:= **linearDomain = {0, 2 Pi}**

Out[18]= {0, 2 π}

In[19]:= **linearFunction = {linearRule, linearDomain}**

Out[19]= {linearRule, {0, 2 π}}

In[20]:= **quadraticRule[x_] := x^2**

In[21]:= **quadraticDomain = {0, 2 * Pi}**

Out[21]= {0, 2 π}

In[22]:= **quadraticFunction = {quadraticRule, quadraticDomain}**

Out[22]= {quadraticRule, {0, 2 π}}

In[23]:= **cubicRule[x_] := x^3**

In[24]:= **cubicDomain = {0, 2 * Pi}**

Out[24]= {0, 2 π}

In[30]:= **cubicFunction = {cubicRule, cubicDomain}**

Out[30]= {cubicRule, {0, 2 π}}

In[31]:= **VectorLength[constantFunction]**

Out[31]= $\sqrt{2\,\pi}$

In[32]:= **VectorLength[linearFunction]**

Out[32]= $2\sqrt{\frac{2}{3}}\ \pi^{3/2}$

In[33]:=
```
Integrate[x * x, {x, 0, 2 Pi}]
```

Out[33]= $\frac{8\,\pi^3}{3}$

In[34]:=
```
VectorLength[quadraticFunction]
```

Out[34]= $4\sqrt{\frac{2}{5}}\;\pi^{5/2}$

In[35]:=
```
VectorLength[cubicFunction]
```

Out[35]= $8\sqrt{\frac{2}{7}}\;\pi^{7/2}$

In[36]:=
```
InnerProduct[quadraticFunction, cubicFunction]
```

Out[36]= $\frac{32\,\pi^6}{3}$

In[37]:=
```
thing = VectorSum[quadraticFunction, cubicFunction]
```

Out[37]= {sumRule$807, {0, 2 π}}

In[38]:=
```
thing[[1]]
```

Out[38]= sumRule$807

In[39]:=
```
sumRule$807[x]
```

Out[39]= $x^2 + x^3$

In[40]:=
```
thing[[2]]
```

Out[40]= {0, 2 π}

In[41]:=
```
ScalarProduct[-1, cubicFunction]
```

Out[41]= {productRule$812, {0, 2 π}}

In[42]:=
```
productRule$812[x]
```

Out[42]= $-x^3$

In[43]:=
```
B = OrthonormalBasis[
  {constantFunction, linearFunction, quadraticFunction, cubicFunction}]
```

Out[43]= {{productRule$1168, {0, 2 π}}, {productRule$1183, {0, 2 π}},
{productRule$1206, {0, 2 π}}, {productRule$1220, {0, 2 π}}}

In[44]:=
```
B[[1]]
```

Out[44]= {productRule$1168, {0, 2 π}}

In[45]:=
```
productRule$1168[x]
```

Out[45]= $\frac{1}{\sqrt{2\,\pi}}$

In[46]:= **B[[2]]**

Out[46]= {productRule$1183, {0, 2 π}}

In[47]:= **productRule$1183[x]**

Out[47]= $\frac{\sqrt{\frac{3}{2}}\ (-\pi + x)}{\pi^{3/2}}$

In[48]:= **B[[3]]**

Out[48]= {productRule$1206, {0, 2 π}}

In[49]:= **productRule$1206[x]**

Out[49]= $\frac{3\sqrt{\frac{5}{2}}\ \left(-\frac{4\pi^2}{3} + x^2 - 2\pi\ (-\pi + x)\right)}{2\pi^{5/2}}$

In[50]:= **B[[4]]**

Out[50]= {productRule$1220, {0, 2 π}}

In[51]:= **productRule$1220[x]**

Out[51]= $\frac{5\sqrt{\frac{7}{2}}\ \left(-2\pi^3 + x^3 - \frac{18}{5}\pi^2\ (-\pi + x) - 3\pi\left(-\frac{4\pi^2}{3} + x^2 - 2\pi\ (-\pi + x)\right)\right)}{2\pi^{7/2}}$

In[52]:= **cosRule[x_] := Cos[x]**

In[53]:= **cosDomain = {0, 2 * Pi}**

Out[53]= {0, 2 π}

In[54]:= **cosFunction = {cosRule, cosDomain}**

Out[54]= {cosRule, {0, 2 π}}

In[55]:= **B**

Out[55]= {{productRule$1168, {0, 2 π}}, {productRule$1183, {0, 2 π}},
{productRule$1206, {0, 2 π}}, {productRule$1220, {0, 2 π}}}

In[56]:= **ProjectionVector[cosFunction, B]**

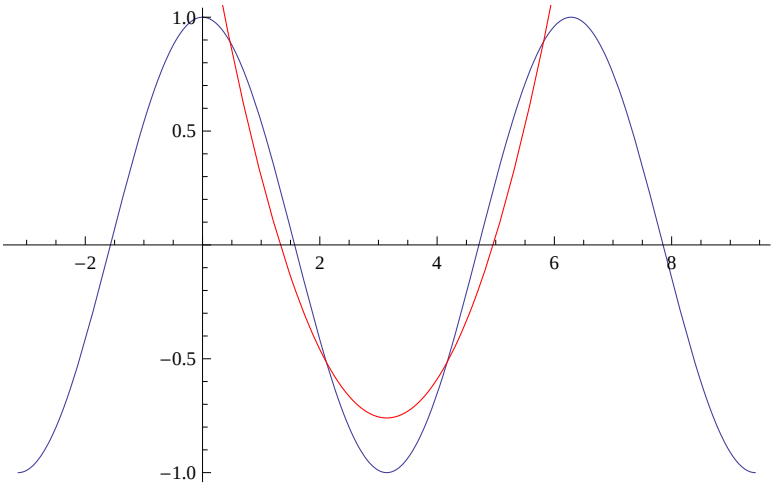
Out[56]= {sumRule$1644, {0, 2 π}}

In[59]:= **Simplify[N[sumRule$1644[x]]]**

Out[59]= $1.51982 - 1.45132\,x + 0.230985\,x^2$

In[60]:=
```
Show[Plot[Cos[x], {x, -Pi, 3 * Pi}],
 Plot[sumRule$1644[x], {x, -Pi, 3 * Pi}, PlotStyle → Red]]
```

Out[60]=



In[61]:=
```
sinRule[x_] := Sin[x]
```

In[62]:=
```
sinDomain = {0, 2 * Pi}
```

Out[62]= {0, 2 π}

In[63]:=
```
sinFunction = {sinRule, sinDomain}
```

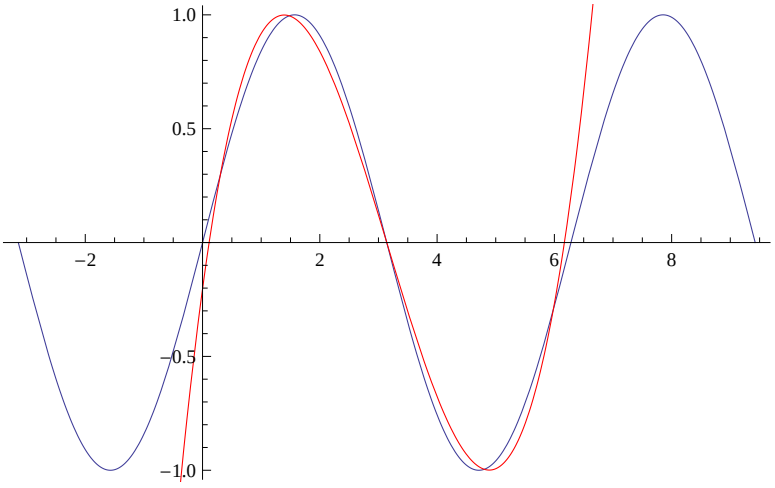
Out[63]= {sinRule, {0, 2 π}}

In[64]:=
```
ProjectionVector[sinFunction, B]
```

Out[64]= {sumRule$2216, {0, 2 π}}

In[65]:=
```
Show[Plot[Sin[x], {x, -Pi, 3 * Pi}],
 Plot[sumRule$2216[x], {x, -Pi, 3 * Pi}, PlotStyle → Red]]
```

Out[65]=



In[66]:=
```
N[sumRule$2216[x]]
```

Out[66]=
```
-0.303964 (-3.14159 + x) + 0.0933877 (-62.0126 -
    35.5306 (-3.14159 + x) + x^3 - 9.42478 (-13.1595 - 6.28319 (-3.14159 + x) + x^2))
```

In[67]:= **Simplify[%]**

Out[67]= $-0.203312 + 1.90812\,x - 0.880158\,x^2 + 0.0933877\,x^3$