Evaluating a double integral by means of iterated integral in polar coordinates

Area in plane corresponding to polar coordinates $r_1 \leq r \leq r_2$ and $\theta_1 \leq \theta \leq \theta_2$: total area between circles is $\pi(r_2^2 - r_1^2)$. Fraction of this area represented is $\frac{\theta_2 - \theta_1}{2\pi}$. Product is $\frac{1}{2}(r_1 + r_2)\Delta_r \Delta_\theta$.

Now consider evaluating $\int\int_A f(x, y)\ dA$ where $\theta_1 \leq \theta \leq \theta_2$ and $r_1 \leq \theta \leq r_2$. Approximate mass of region above is $f(r^* \cos\theta^*, r^* \sin\theta^*) r^* \Delta_r \Delta_\theta$, where r^* and θ^* are the midpoints. Add up these regions and get Riemann sum for the double integral

$$\int\int_{\substack{\theta_1 \leq \theta \leq \theta_2 \\ r_1 \leq \theta \leq r_2}} r f(r\cos\theta, r\sin\theta)\ dA.$$

This yields formula for polar coordinates evaluation.