

Center of Mass in Two Dimensions

Let R be a region of the plane. Center of mass is average position in the region. To approximate, let $f(x, y)$ be mass density function. The fraction of mass in region R_i is approximately

$$\frac{f(x_i, y_i)A(R_i)}{\int \int_R f(x, y) dA}.$$

The fraction of the x -coordinate is

$$\frac{x_i f(x_i, y_i)A(R_i)}{\int \int_R f(x, y) dA}.$$

Summing and taking the limit, we obtain

$$\bar{x} = \frac{\int \int_R x f(x, y) dA}{\int \int_R f(x, y) dA}.$$

Similarly, we have

$$\bar{y} = \frac{\int \int_R y f(x, y) dA}{\int \int_R f(x, y) dA}.$$