Week 9 Lectures Math 223

Section 16.1: Multiple-Sigma Notation

Iterated sums. Can be factored. $\Delta x_i = (x_i - x_{i-1})$. In problem 11, use $(a+b)(a-b) = a^2 - b^2$ formula etc.

Section 16.2: Double Integrals

Review of single-variable definite integral over an interval: chop interval into subintervals. Find minimum value and maximum value of function on each interval. Compute products and sum. Get lower and upper approximations. Both approach each other as number of subintervals increases. Take limit. Interpret as area.

Generalize to two variable functions defined over a rectangular region: chop into subrectangles. Find minimum function value and maximum function value. Compute products of function value with area of subrectangles and sum. Get lower and upper approximations. Both approach each other as number of subrectangles increases. Take limit. Interpret as volume.

A useful technique: if lower sum includes x_{i-1} and upper sum includes x_i, then an intermediate value can be taken to be

$$\frac{1}{2}(x_{i-1} + x_i).$$

Now we can exploit $(a-b)(a+b) = a^2 - b^2$ formula. Similarly, between x_{i-1}^2 and x_i^2 there is

$$\frac{1}{3}(x_{i-1}^2 + x_{i-1}x_i + x_i^2).$$

Exploit $(a-b)(a^2 + ab + b^2) = a^3 - b^3$ formula.

Double integral over a non-rectangular bounded region: Extend function to bounding rectangle.

Mean Value Theorem for Double Integrals: skip this.