Math 223 Week 1 Lectures: Sections 10.6, 10.7, 10.8

Section 10.6: Cylinders and Quadric Surfaces

Graph of an equation in 2 dimensions: plot of all (x, y) that satisfy equation. Example: $x^2 + y^2 = 25$.

Graph of an equation in 3 dimensions: plot of all (x, y, z) that satisfy equation. Example: $x^2 + y^2 + z^2 = 169$ – one coordinate is $(3, 4, 5)$.

When you graph an equation in 2 variables and 3 dimensions, you get a cylinder. Examples: $x^2 + y^2 = 25$, $z = y^2$, $x - 2z = 0$.

Quadric surfaces are plots of a general quadratic equation – page 554.

How to plot: superimpose traces (equation where one variable is fixed at particular number). For example, $y^2 = x^2 + z^2$.

Classification: Table 1, page 557.

Example: $4x^2+y^2+4z^2-4y-24z+36 = 0$ – requires completing the square, consulting the table, recognizing shifts.

Section 10.7: Vector Functions and Space Curves

A vector function is $r(t) = (f(t), g(t), h(t)) = f(t)i + g(t)j + h(t)k$. t can be viewed as time, $(f(t), g(t), h(t))$ as position at time t.

Example: Line through $(1, 2, 3)$ and $(10, 9, 8)$.

Example: Intersection of $x^2 + y^2 = 1$ and $y + z = 2$. Several ways to parameterize: $t =$ angle with x-axis, $t = z$-coordinate.

Derivative of a vector function: differentiate the components.

Geometric interpretation: $r'(t) =$ direction of motion at time t. Determines tangent line direction.

Tangent line equation at time t_0: $L(t) = r(t_0) + tr'(t_0)$.

Work out an example.

Differentiation rules are given on page 566.

The definite integral is defined on page 567.

Section 10.8: Arc Length and Curvature

The arclength formula is given on page 570. Idea of derivation: we learn that $s = \int v$ in calculus I. What is v in three dimensions?

$$v = \lim_{h \to 0} \frac{||r(t+h) - r(t)||}{h} = \lim_{h \to 0} \left\| \frac{\|r(t+h) - r(t)}{h} \right\| \| = ||r'(t)||.$$

Arclength as a function of time: formula 6, page 571.

Position as a function of arclength: solve for t in terms of s, then define $R(s) = r(t) = \cdots$. Example: $r(t) = (t^3, t^3, t^3)$, $R(s) = (\frac{s}{\sqrt{3}}, \frac{s}{\sqrt{3}}, \frac{s}{\sqrt{3}})$.

Unit tangent vector: tangent vector scaled down to length 1. Regard as a length one gauge in your airplane which points in the direction of travel.

Curvature: if the airplane does a sharp turn, you would expect to see the unit tangent vector change. The more it changes, the sharper the turn. Mathematical definition is curvature, κ. Formula:

$$\kappa = \left|\left| \frac{dT}{ds} \right|\right| = \frac{||T'(t)||}{||r'(t)||}.$$

An interpretation in the plane: Gauge direction is determined by angle.

$$\kappa = \left|\left| \frac{dT}{ds} \right|\right| = \lim_{h \to 0} \frac{||T(s+h) - T(s)||}{h} =$$

$$\lim_{h \to 0} \left| \frac{\theta(s+h) - \theta(s)}{h} \right| = \text{rate of change of } |\text{angle}| \text{ with respect to arclength.}$$

Alternative formulas: Theorem 10, Theorem 11. Work out some examples.

Normal and Binormal vectors: see top of page 575 for formulas. $N(t)$ is perpendicular to the direction of motion and has length 1. The binormal is perpendicular to the plane containing $T(t)$ and $N(t)$, i.e. $r(t)$ and $r'(t)$, and has length 1. This plane is called the osculating plane. The osculating circle is illustrated on page 576 and has equation

$$osc(t) = r(t_0) + \frac{1}{\kappa(t_0)} N(t_0) + \frac{\cos(t)}{\kappa(t_0)} T(t_0) + \frac{\sin(t)}{\kappa(t_0)} N(t_0).$$

Work out some examples.