Antiderivatives

Definition.

Examples.

Distance is antiderivative of velocity, velocity is antiderivative of acceleration.

Motion problems.

Areas and Distances

Area below graph of $y = f(x)$ is approximated by Riemann sums and can be defined as the limit of these expressions.

Example: a limit argument shows that the area below $y = x^2$ over $[0, 1]$ is $\frac{1}{3}$.

Application: $f(b) - f(a)$ can be viewed as area below $y = f'(x)$ between $x = a$ and $x = b$. Reason: using the mean value theorem applied to the partition $a = x_0 < x_1 < \cdots < x_n = b$, we obtain

$$f(b) - f(a) = f(x_1) - f(x_0) + f(x_2) - f(x_1) + \cdots =$$

$$f'(c_1)(x_1 - x_0) + f'(c_2)(x_2 - x_1) + \cdots =$$

Riemann sum!

Given that every partition of $[a, b]$ produces $f(b) - f(a)$ as a possible Riemann sum, the limit of these sums has to be $f(b) - f(a)$.

Application: Net distance travelled can be viewed as area below the velocity curve.

Application: Let $f(x) = \frac{1}{3}x^3$. Then $\frac{1}{3} = f(1) - f(0)$ is the area below $y = f'(x) = x^2$ between $x = 0$ and $x = 1$.