Homework 9 due Thursday, November 13:

Pages 87–88, problems 1.5, 1.11, 1.13, 1.18, 1.20 and pages 95–96, problems 2.6, 2.7

Hints:

1.5: Given $f_0 + f_1 u + \cdots + f_n u^n \in K[u]$ where each $f_i \in K$ and given $\tau \in \mathrm{Gal}_K(K[u])$, compute $\tau(f_0 + f_1 u + \cdots + f_n u^n)$.

1.11: Refer back to exercise [1.5]. Given $\tau \in \mathrm{Gal}_{\mathbb{Q}}(\sqrt[3]{5})$, show that $\tau(\sqrt[3]{5})^3 = 5$. Given that $\tau(\sqrt[3]{5}) \in \mathbb{Q}[\sqrt[3]{5}]$, what can it be equal to?

1.13: Instead of following the method of Example [1.25.2], use the Galois Group Algorithm presented in class (see class website for a statement of the algorithm).

1.18: A primitive p^{th} root of unity is any complex number ξ that satisfies $\xi^p = 1$ and $\xi^r \neq 1$ for $0 \leq r < p$. Note also that $(x-1)f(x) = x^p - 1$. Use the Galois Group Algorithm and exercise [1.5] to compute $\mathrm{Gal}_{\mathbb{Q}}(\mathbb{Q}[\xi])$.

1.20: Use the Galois Group Algorithm.

2.6: Consider the towers $F \subseteq F[u] \subseteq F[u,v]$ and $F \subseteq F[v] \subseteq F[u,v]$ and use the Tower Theorem.

2.7: (a) First establish $[F : \mathbb{Q}] = p(p-1)$ using exercise [2.6], then use this information to find the irreducible polynomial of ξ over $\mathbb{Q}$ and ω over $\mathbb{Q}[\xi]$. (b) The conjugates of an element $u \in E$ in a field extension $F \subseteq E$ are the roots of $\mathrm{irr}(u, F)$ that belong to E. (c) Construct all the elements in $\mathrm{Gal}_{\mathbb{Q}}(\mathbb{Q}[\xi, \omega])$ using the Galois Group Algorithm. Once you have constructed it, you can determine how many group elements there are.