Homework 7 due Thursday, October 23:

Pages 71–72, problems 5.6, 5.8, 5.9, 5.10, 5.11, 5.13, 5.17
Hints:

5.6: Plug in, use long division.

5.8. Use Eisenstein's criterion and the material developed in class.

5.9. F_2 is the same as $\mathbb{Z}_2$. (c) If a fifth degree polynomial is reducible, then it has a factor of degree ≤ 2. (d) The elements of $F_2[x]/(f(x))$ are cosets of the form $(f(x)) + g(x)$ where $g(x) \in F_2[x]$. Using long division, $g(x) = q(x)f(x)+r(x)$ where $\deg r(x) < 5$. Explain why this implies $(f(x))+g(x) = (f)+r(x)$. Hence every element in $F_2[x]/(f(x))$ is of the form $(f)+a+bx+cx^2+dx^3+ex^4$ where $a, b, c, d, e \in F_2$. Show that the representation is unique, i.e. $(f)+a+bx+cx^2+dx^3+ex^4 = (f)+A+Bx+Cx^2+Dx^3+Ex^4$ implies $a = A, b = B, c = C, d = D, e = E$, then count the elements of the quotient ring using this representation.

5.13. (a) is reducible!

17. By Corollary [5.28], $1 + x + \cdots + x^{n-1}$ is irreducible if n is prime. You must show that if $n = ab$ where $a > 1$ and $b > 1$ then $1 + x + \cdots + x^{n-1}$ can be factored non-trivially.