Homework 4 due Thursday, October 2.

Page 33, problems 2.20, 2.23. Page 42, problems 3.1, 3.2, 3.6, 3.8, 3.9, 3.11.

Remarks:

2.20. The class equation is $|G| = |Z(G)| + \sum_{i=k+1}^{m} \frac{|G|}{|G_{x_i}|}$ where the distinct conjugacy classes of G are $[x_1], \ldots, [x_m]$, $x_1, \ldots, x_k \in Z(G)$, and $G_x = \{g \in G : gxg^{-1} = x\}$. So you must compute the conjugacy class of every element in D_4, choose $x_1, \ldots, x_m$ (one from each class), then compute each G_{x_i}, then check that the class equation holds for D_4.

2.23. You are given that p does not divide $|G|/|P|$ and must show that p does not divide $|N|/|N \cap P|$. You will find Corollary [2.18], p. 23, useful.

3.1. By the Fundamental Theorem of Finite Abelian Groups, each such group is isomorphic to a direct product of cyclic groups of the form $\mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2} \times \cdots$. You should give a reason why the abelian groups of order 72 you produce are non-isomorphic. See Example 1, p. 38.

3.2. Consider the product of two groups, one of them non-abelian. Describe the group operation carefully.

3.6. Use contradiction.

3.8. Prove that hint given in the textbook is correct: each subgroup is normal in the next, and the quotient groups are abelian.

3.9. Find an explicit solvable series for D_n for an arbitrary n and prove that it has the desired properties.