In[1]:= $f[1] = 4 + 3 x + 6 x^2 + 2 x^3 + 5 x^4 + x^7; g[1] = 4 + 6 x + 8 x^2 + 5 x^3 + 10 x^5 + 7 x^6 + x^8;$

In[2]:= $\{f[1], g[1]\}$

Out[2]= $\left\{4 + 3 x + 6 x^2 + 2 x^3 + 5 x^4 + x^7, \ 4 + 6 x + 8 x^2 + 5 x^3 + 10 x^5 + 7 x^6 + x^8\right\}$

In[3]:= $\{f[2], g[2]\} = \text{PolynomialMod}[\text{Expand}[\{f[1], g[1] - x\,f[1]\}], 11]$

Out[3]= $\left\{4 + 3 x + 6 x^2 + 2 x^3 + 5 x^4 + x^7, \ 4 + 2 x + 5 x^2 + 10 x^3 + 9 x^4 + 5 x^5 + 7 x^6\right\}$

In[4]:= $\{f[3], g[3]\} = \text{PolynomialMod}[\text{Expand}[\{f[2] - 8 x\,g[2], g[2]\}], 11]$

Out[4]= $\left\{4 + 4 x + x^2 + 6 x^3 + 2 x^4 + 5 x^5 + 4 x^6, \ 4 + 2 x + 5 x^2 + 10 x^3 + 9 x^4 + 5 x^5 + 7 x^6\right\}$

In[5]:= $\{f[4], g[4]\} = \text{PolynomialMod}[\text{Expand}[\{f[3] - 10\,g[3], g[3]\}], 11]$

Out[5]= $\left\{8 + 6 x + 6 x^2 + 5 x^3 + 10 x^5, \ 4 + 2 x + 5 x^2 + 10 x^3 + 9 x^4 + 5 x^5 + 7 x^6\right\}$

In[6]:= $\{f[5], g[5]\} = \text{PolynomialMod}[\text{Expand}[\{f[4], g[4] - 4 x\,f[4]\}], 11]$

Out[6]= $\left\{8 + 6 x + 6 x^2 + 5 x^3 + 10 x^5, \ 4 + 3 x + 3 x^2 + 8 x^3 + 5 x^5\right\}$

In[7]:= $\{f[6], g[6]\} = \text{PolynomialMod}[\text{Expand}[\{f[5], g[5] - 6\,f[5]\}], 11]$

Out[7]= $\left\{8 + 6 x + 6 x^2 + 5 x^3 + 10 x^5, \ 0\right\}$

In[8]:= $\text{delta} = \text{PolynomialMod}[10\,f[6], 11]$

Out[8]= $3 + 5 x + 5 x^2 + 6 x^3 + x^5$

In[9]:= $\text{PolynomialMod}[\text{PolynomialQuotient}[f[1], \text{delta}, x], 11]$

Out[9]= $5 + x^2$

In[10]:= $\text{PolynomialRemainder}[f[1], \text{delta}, x]$

Out[10]= $22 + 33 x + 33 x^2 + 33 x^3$

In[11]:= $\text{PolynomialMod}[\text{PolynomialQuotient}[g[1], \text{delta}, x], 11]$

Out[11]= $5 + x + x^3$

In[12]:= $\text{PolynomialRemainder}[g[1], \text{delta}, x]$

Out[12]= $-11 - 22 x - 22 x^2 - 33 x^3 - 11 x^4$

In[13]:= $F[2] = F[1]; G[2] = G[1] - x\,F[1]; F[3] = F[2] - 8 x\,G[2]; G[3] = G[2]; F[4] = F[3] - 10\,G[3];$
$G[4] = G[3]; F[5] = F[4]; G[5] = G[4] - 4 x\,F[4]; F[6] = F[5]; G[6] = G[5] - 6\,F[5];$

In[14]:= $\text{PolynomialMod}[\text{Simplify}[F[6]], 11]$

Out[14]= $F[1] + 10 x\,F[1] + 8 x^2\,F[1] + G[1] + 3 x\,G[1]$

In[15]:= $a = 1 + 10 x + 8 x^2; b = 1 + 3 x; \text{PolynomialMod}[\text{Simplify}[a * f[1] + b * g[1]], 11]$

Out[15]= $8 + 6 x + 6 x^2 + 5 x^3 + 10 x^5$

In[16]:= $\text{PolynomialMod}[\text{PolynomialRemainder}[\text{delta}, x^2 + 5, x], 11]$

Out[16]= 0

In[17]:= **PolynomialQuotient[delta, x^2 + 5, x]**

Out[17]= $5 + x + x^3$