

Math 492-01: Math Capstone Experience  
Mon-Wed-Thu 1:00 pm - 1:50 pm  
Classroom: TR C114  
Office: Wissink 263

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Office Hours: MWH 2-3, T 1-3

**From the Undergraduate Bulletin:** This course is designed to allow the undergraduate student an opportunity to integrate their undergraduate mathematics experiences by engaging the student in working on a problem in applied or theoretical mathematics. In doing so, the student will see the connections among the various topics found in the undergraduate mathematics curriculum. Content will vary by semester. Problems will arise from the course content and materials as presented by the instructor. An important component of this course will be the preparation and presentation of a research paper describing the student's progress toward a solution of the problem under consideration. Because of the breadth of mathematical topics needed for successful completion of the course, students need to have senior standing. Pre: Two of the following: MATH 316, MATH 345, MATH 375 and senior standing (or permission of the instructor).

**The Mathematics Capstone Experience: What is it?** The idea is to give students an opportunity to put their mathematics education to use in some organic context, working on a problem or set of problems that span different subjects, and practice mathematical communication skills both oral and written. A kind of finishing school for math majors wherein they go beyond narrow homework sets and engage in the type of mathematical activity common to teachers, researchers, and other practitioners. In the course this semester we will see a blend of number theory, abstract algebra, linear algebra, and discrete mathematics.

**Agenda for the Semester:** We will study selected topics in Algebraic Number Theory. A typical result that we will prove is that every positive integer can be expressed as the sum of four squares. We will start with the properties of the integers that students learn in Math 290, for example unique factorization into primes, and generalize to other number systems. I will give a series of lectures on this subject and ask that students take careful notes and ask questions as necessary. I will also assign problems from the book "Elements of Number Theory" by John Stillwell (Springer-Verlag, 2003). The textbook has a number of optional sections which I will assign to students as miniature research projects. Each student will choose one research topic, submit a written report, and present their findings to the class in an oral presentation.

**Highlights of the Course:** The Euclidean algorithm applied to ordinary integers, algebraic integers in certain quadratic fields, and the ring of Hurwitz integers; solving Diophantine equations using unique factorization; the tree of relatively prime pairs; Conway's combinatorial description of the outputs of a binary quadratic form; the two-squares theorem; the four-squares theorem; the eight-squares theorem; the Chinese remainder theorem and its use in proving the multiplicativity of Euler's totient function; quadratic reciprocity; unique factorization of ideals into prime ideals in rings of quadratic algebraic numbers; characterization of primes of the form  $x^2+ny^2$  for certain values of  $n$ .

**Evaluation and Grades:** Students will be required to take notes, do homework problems, and submit both written and oral reports on a project of their choosing. Notes are worth 100 points, homework is worth 100 points, the written report is worth 100 points, and the oral report is worth 100 points, for a total of 400 points. Students will receive points based on the quality of their work. Grades are based on a straight scale: 90% (360 points) for an A, 80% (320 points) for a B, etc. Failure to turn in a category of work (notes, homework problems, written report, oral report) will result in a failing grade.

**Grade Policy:** Your grade is based on your performance during the 16 weeks of the semester in accordance with the grade calculation above. I will not change any grades after they have been submitted to the Registrar, and I will not consent to extra-credit opportunities designed for the express purpose of raising the grade of one individual.

**Attendance Policy:** Attendance is mandatory. Excessive absences will result in a failing grade.

**Student Conduct:** Please arrive to class on time, do not talk unless making a contribution to the class, and stay for the entire class period. Use of electronic devices (smart phones, laptops etc.) is forbidden.

**Projects:** See handout.

**Important Dates:**

September 1, November 27: No classes scheduled.

October 30: Students must pick a project by this date.

November 13: First draft of written report due.

Week of November 24 and December 1: Oral presentations of projects by students.

December 9: Written reports due.