

In[12]:= **xi = E ^ (2 Pi I / 3) ;**

In[13]:= **r = 2 ^ (1 / 3) ;**

In[14]:= **MyProduct[a_, b_, c_] := Simplify[
 (a + b*r + c*r^2) (a + b*r*xi + c*r^2*xi^2) (a + b*r*xi^2 + c*r^2*xi)]**

In[15]:= **MyProduct[a, b, c]**

Out[15]= $a^3 + 2 b^3 - 6 a b c + 4 c^3$

In[16]:= **MyExpression[a_, b_, c_] := a + b*r + c*r^2**

In[17]:= **MyInverse[a_, b_, c_] := Expand[Simplify[(a + b*r*xi + c*r^2*xi^2)
 (a + b*r*xi^2 + c*r^2*xi) / (a^3 + 2 b^3 - 6 a b c + 4 c^3)]]**

In[18]:= **MyInverse[a, b, c]**

Out[18]=
$$\frac{a^2}{a^3 + 2 b^3 - 6 a b c + 4 c^3} - \frac{2^{1/3} a b}{a^3 + 2 b^3 - 6 a b c + 4 c^3} + \frac{2^{2/3} b^2}{a^3 + 2 b^3 - 6 a b c + 4 c^3} -$$
$$\frac{2^{2/3} a c}{a^3 + 2 b^3 - 6 a b c + 4 c^3} - \frac{2 b c}{a^3 + 2 b^3 - 6 a b c + 4 c^3} + \frac{2 \times 2^{1/3} c^2}{a^3 + 2 b^3 - 6 a b c + 4 c^3}$$

In[19]:= **MyExpression[a, b, c] * MyInverse[a, b, c]**

Out[19]=
$$(a + 2^{1/3} b + 2^{2/3} c) \left(\frac{a^2}{a^3 + 2 b^3 - 6 a b c + 4 c^3} - \frac{2^{1/3} a b}{a^3 + 2 b^3 - 6 a b c + 4 c^3} + \frac{2^{2/3} b^2}{a^3 + 2 b^3 - 6 a b c + 4 c^3} - \right.$$
$$\left. \frac{2^{2/3} a c}{a^3 + 2 b^3 - 6 a b c + 4 c^3} - \frac{2 b c}{a^3 + 2 b^3 - 6 a b c + 4 c^3} + \frac{2 \times 2^{1/3} c^2}{a^3 + 2 b^3 - 6 a b c + 4 c^3} \right)$$

In[20]:= **Simplify[%]**

Out[20]= 1

In[21]:= **p = MyExpression[1, 2, 3]**

Out[21]= $1 + 2 \times 2^{1/3} + 3 \times 2^{2/3}$

In[22]:= **q = MyInverse[1, 2, 3]**

Out[22]= $-\frac{11}{89} + \frac{16 \times 2^{1/3}}{89} + \frac{2^{2/3}}{89}$

In[23]:= **Simplify[p * q]**

Out[23]= 1