

Math 122

Week 9 Solutions to Selected Problems

Section 8.5

21. The ratio of a_{n+1} to a_n is

$$\frac{((n+1)!)^k (kn)!}{(kn+k)!(n!)^k} = \frac{(n+1)^k}{(kn+k)(kn+k-1)\cdots(kn+1)} =$$
$$\frac{(n+1)^k}{k^k (n + \frac{k}{k})(n + \frac{k-1}{k})\cdots(n + \frac{1}{k})} = \frac{1}{k^k} \left(\frac{n+1}{n + \frac{k}{k}}\right) \left(\frac{n+1}{n + \frac{k-1}{k}}\right) \cdots \left(\frac{n+1}{n + \frac{1}{k}}\right).$$

These expressions tend to $\frac{1}{k^k}$ as $n \rightarrow \infty$. Therefore $R = k^k$.