Math 223

Week 8 Solutions to Selected Problems

Section 12.6

25. The cross-sections of the paraboloid $z = 4x^2 + 4y^2$ that are parallel to the xy-plane are circles of radius $\frac{\sqrt{z}}{2}$. The largest one in the paraboloid occurs at $z = a$. For each x and y in this region, z rizes from the surface of the paraboloid to a. So the region bounded by the paraboloid and the plane can be described as all (x, y, z) where $x = r\cos\theta$, $y = r\sin\theta$, $0 \leq \theta \leq 2\pi$, $0 \leq r \leq \frac{\sqrt{a}}{2}$, and $4r^2 \leq z \leq a$. Hence both mass and center of mass can be computed by evaluating integrals of the form

$$\int_0^{2\pi} \int_0^{\frac{\sqrt{a}}{2}} \int_{4r^2}^{a} f(r\cos\theta, r\sin\theta, z) r \; dz \; dr \; d\theta.$$