Section 12.2

17. The base of the region, I assume, is in the xy-plane where $z = 0$. In this region, x varies from 0 to 1, and for a given value of x, y varies from x^4 to x. The height above the point $(x, y, 0)$ is $z = x + 2y$. Putting this all together, the volume is

$$V = \int_0^1 \int_{x^4}^{x} x + 2y \; dy \; dx = \frac{7}{18}.$$

27. This solid intersects the region of the xy-plane bounded by $y = 1 - x^2$ and $y = x^2 - 1$. This can be described as all $(x, y, 0)$ where $-1 \leq x \leq 1$ and, for a given value of x in this range, $x^2 - 1 \leq y \leq 1 - x^2$. (You have to draw these two parabolas and calculate the intersection points to see this.) To estimate the volume of the solid, cut this region into rectangles of dimensions $\Delta x \Delta y$, pick a point (x_i^*, y_i^*) in the i^{th} rectangle, and extend a line through this point, perpendicular to the xy plane, that extends between the two planes $x + y + z = 2$ and $2x + 2y - z + 10 = 0$. To get the approximate volume of the rectangular solid this suggests, multiply the area of the rectangle by the distance between the two z-coordinates. This line intersects the first plane in the point $(x_i^*, y_i^*, 2 - x_i^* - y_i^*)$ and the second plane in the point $(x_i^*, y_i^*, 2x_i^* + 2y_i^* + 10)$, and the distance between these points is $3x_i^* + 3y_i^* + 8$. So the approximate volume is $\sum (3x_i^* + 3y_i^* + 8)\Delta x \Delta y$. The exact volume is $\int\int_R 3x + 3y + 8 \; dA$, where R is $\{(x, y) : -1 \leq x \leq 1, x^2 - 1 \leq y \leq 1 - x^2\}$. So

$$V = \int_{-1}^{1} \int_{x^2-1}^{1-x^2} 3x + 3y + 8 \; dy \; dx = \frac{64}{3}.$$

39. The region of integration is bounded by $y = 0$ and $y = 3$. For a given value of y, x ranges from y^2 to 9. Graphing this information we can see that the region of integration is bounded by $y = 0$, $y = \sqrt{x}$, $x = 0$ and $x = 9$. So the type I description of this region is $0 \leq x \leq 9$, $0 \leq y \leq \sqrt{x}$. Hence the double integral is equal to

$$\int_0^9 \int_0^{\sqrt{x}} y \cos(x^2) \; dy \; dx = \int_0^9 \frac{y^2}{2} \cos(x^2) \Big|_0^{\sqrt{x}} \; dx =$$

$$\frac{1}{2} \int_0^9 x \cos(x^2) \; dx.$$

Now use the u-substitution $u = x^2$.