Selected Solutions to Homework 3 Problems

Section 11.3

33. $f(x,y) = xy^2 - x^3y$,

$$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h} = \lim_{h\to 0} \frac{(x+h)y^2 - (x+h)^3y - xy^2 - x^3y}{h} =$$

$$\lim_{h\to 0} \frac{xy^2 + hy^2 - x^3y - 3x^2hy - 3xh^2y - h^3y - xy^2 + x^3y}{h} =$$

$$\lim_{h\to 0} y^2 - 3x^2y - 3xhy - h^2y = y^2 - 3x^2y$$

45. $f = x(x+y)^{-1}$, $f_x = (x+y)^{-1} + x(-1)(x+y)^{-2} = (x+y)^{-1} - x(x+y)^{-2}$, $f_{xy} = (f_x)_y = (-1)(x+y)^{-2} - x(-2)(x+y)^{-3} = -(x+y)^{-2} + 2x(x+y)^{-3}$.

64. Rate of change in the y direction is $T_y(2,1)$. We have $T = 60(1 + x^2 + y^2)^{-1}$, $T_y = 60(-1)(1 + x^2 + y^2)^{-2}(2y) = -120y(1 + x^2 + y^2)^{-2}$, hence $T_y(2,1) = -120(6)^{-2} = -120/36$ degrees per meter.

73. The ellipsoid $4x^2 + 2y^2 + z^2 = 16$ intersects the plane $y = 2$ at all points $(x, 2, z)$ where $4x^2 + z^2 = 8$. So the tangent line will be in the plane $y = 2$ and tangent to the curve $4x^2 + z^2 = 8$. Now let $r(t) = (x(t), 2, z(t))$ be a trajectory along this ellipse. Its tangent vector when it passes through the point $(1, 2, 2)$ will provide the direction vector for the tangent line. Note that $4x^2 + z^2 = 8$ implies $z = \sqrt{8 - 4x^2}$ above the xy plane. So a possible formula for $r(t)$ is

$$r(t) = (t, 2, \sqrt{8 - 4t^2}).$$

This passes through $(1, 2, 2)$ at time $t = 1$, so the direction of motion at time $t = 1$ is $r'(1)$ and the tangent line has parameterization $(1, 2, 2) + tr'(1)$. Working out the details,

$$r'(t) = (1, 0, \frac{1}{2}(8 - 4t^2)^{\frac{-1}{2}}(-8t)) = (1, 0, \frac{-4t}{\sqrt{8 - 4t^2}})$$

$$r'(1) = (1, 0, -2).$$

Therefore the tangent line is $(1, 2, 2) + t(1, 0, -2) = (1 + t, 2, 2 - 2t)$ with $x(t) = 1 + t$, $y(t) = 2$, $z(t) = 2 - 2t$.

Section 11.4

19. $z = x^3 \ln(y^2) = 2x^3 \ln y$, $dz = z_x\ dx + z_y\ dy = 6x^2 \ln y\ dx + \frac{2x^3}{y}\ dy$.

23. $\Delta z = 5(1.05)^2 + (2.1)^2 - 5(1)^2 - 2^2 = 0.9225$, $dz = z_x\ dx + z_y\ dy = 10x\ dx + 2y\ dy = 10(.05) + 4(.1) = 0.9$.