Math 223

Week 10 Solutions to Selected Problems

Section 13.1

32. Let $r(t) = (x(t), y(t))$ have a velocity vector dictated by the vector field $F(x, y) = (1, x)$. This means

$$(x'(t), y'(t)) = F(x(t), y(t)) = (1, x(t)).$$

Therefore $x'(t) = 1$ and $y'(t) = x(t)$. This implies the system of differential equations $\frac{dx}{dt} = 1$ and $\frac{dy}{dt} = x$. We must have $x = t + C$ for some C, therefore $\frac{dy}{dt} = t + C$, which implies $y = \frac{1}{2}t^2 + Ct + D$ for some D. If the particle passes through the origin at time $t = 0$, then we have $C = D = 0$, which yields $x = t$ and $y = \frac{1}{2}t^2$. Hence $y = \frac{1}{2}x^2$ is the equation of the curve traced out by $r(t)$. Another way to derive this:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{1} = x,$$

therefore $dy = x\ dx$, therefore $\int dy = \int x\ dx$, therefore $y = \frac{x^2}{2} + C$. If the curve passes through the origin then $C = 0$ and $y = \frac{1}{2}x^2$.