Selected Solutions to Homework 1 Problems

Section 10.6

21. $z^2 = 4x^2 + 9y^2 + 36$ has the same solution set as $-\frac{x^2}{3^2} - \frac{y^2}{2^2} + \frac{z^2}{1^2} = 1$. This is a Hyperboloid of Two Sheets.

29. $z = \sqrt{x^2 + y^2}$ is a cone with vertex at the origin and with the positive z axis as its axis of symmetry. $x^2 + y^2 = 1$ with $1 \leq z \leq 2$ is a cylinder of radius 1 with the z axis between 1 and 2 as its axis of symmetry. The region bounded between the two is outside the cylinder and inside the cone between $z = 1$ and $z = 2$.

34. The curve of intersection is the set of (x, y, z) which satisfy both equations simultaneously. Subtracting 2 times the second equation from the first, we obtain $-6x - 5y = -2$, which is the equation of a plane.

Section 10.7

33. Given that $x(t) = t - 2$ and $y(t) = t^2 + 1$, we can see that $y = (x+2)^2 + 1$ at all points on the curve. This is a parabola opening upward with vertex at $(-2, 1)$. Draw $r'(t)$ as a vector displacing from $r(t)$ at each point $r(t)$ on the curve. It should be tangent to the curve at each point.

46. $r(t) = (\cos t)i + (3t)j + (2\sin 2t)k$, $r'(t) = (-\sin t)i + 3j + (4\cos 2t)k$, $r'(0) = 0i + 3j + 4k$, $T(0) = \frac{r'(0)}{|r'(0)|} = \frac{0i+3j+4k}{\sqrt{0^2+3^2+4^2}} = 0i + \frac{3}{5}j + \frac{4}{5}k$.

49. Tangent line at time t_0 has equation $L(t) = r(t_0) + r'(t_0)t$. Here $r(t) = (t^5, t^4, t^3)$, $r'(t) = (5t^4, 4t^3, 3t^2)$ and $t_0 = 1$, therefore

$$L(t) = (1, 1, 1) + (5, 4, 3)t = (1 + 5t, 1 + 4t, 1 + 3t).$$

55. Let θ be the angle between $r_1'(0)$ and $r_2'(0)$. Then

$$r_1'(0) \cdot r_2'(0) = |r_1'(0)||r_2'(0)| \cos\theta,$$

therefore

$$\theta = \cos^{-1} \frac{r_1'(0) \cdot r_2'(0)}{|r_1'(0)||r_2'(0)|} = \cos^{-1} \frac{(1, 0, 0) \cdot (1, 2, 1)}{1\sqrt{6}} = \cos^{-1} \frac{1}{\sqrt{6}} \approx 66^\circ.$$

58. $\int_0^1 \left(\frac{4}{1+t^2}j + \frac{2t}{1+t^2}k\right)\, dt = (4\tan^{-1} t)j + (\ln(1+t^2))k|_0^1 = 4\frac{\pi}{4}j + (\ln 2)k - (4\cdot 0j + 0\cdot k) = \pi j + (\ln 2)k$. The j part of the integral requires the trig substitution $t = \tan\theta$ and the k part of the integral requires the u-substitution $u = 1 + t^2$.

Section 10.8

14.

$$T(t) = \left(\frac{1}{\sqrt{5t^2+1}}, \frac{t}{\sqrt{5t^2+1}}, \frac{t}{\sqrt{5t^2+1}}\right)$$

$$N(t) = \left(-\frac{\sqrt{5}t}{\sqrt{5t^2+1}}, \frac{1}{\sqrt{25t^2+5}}, \frac{2}{\sqrt{5t^2+5}}\right)$$

$$\kappa(t) = \frac{\sqrt{5}}{(5t^2+1)^{3/2}}$$

37. The plane normal to (a, b, c) passing through (x_0, y_0, z_0) has equation $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$. The normal plane has normal T and the osculating plane has normal $B = T \times N$. Now $r(t) = (2\sin 3t, t, 2\cos 3t)$, $r'(t) = (6\cos 3t, 1, -6\sin 3t)$, $T(t) = \frac{r'(t)}{|r'(t)|} = \frac{1}{\sqrt{7}}(6\cos 3t, 1, -6\sin 3t)$, $T'(t) = \frac{1}{\sqrt{7}}(-18\sin 3t, 0, -18\cos 3t)$, $N(t) = \frac{T'(t)}{|T'(t)|} = (-\sin 3t, 0, -\cos 3t)$, $B(t) = T(t) \times N(t) = \frac{1}{\sqrt{7}}(-\cos 3t, 6, \sin 3t)$. At time $t = \pi$ we arrive at the point $(0, \pi, -2)$. At this time we have $T(\pi) = \frac{1}{\sqrt{7}}(-6, 1, 0)$ and $B(\pi) = \frac{1}{\sqrt{7}}(1, 6, 0)$, therefore

$$\begin{aligned} \text{normal plane:} \quad & -6x + (y - \pi) = 0, \\ \text{osculating plane:} \quad & x + 6(y - \pi) = 0. \end{aligned}$$

41. At the bottom of page 575 is a definition of the normal plane to a curve C at the point P: it is the set of all vectors originating from the point P which are perpendicular to the tangent vector T. Therefore the normal plane is perpendicular to T. We are told that this plane is parallel to the plane $6x + 6y - 8z = 1$, therefore T is perpendicular to $6x + 6y - 8z = 1$. The latter has normal vector $(6, 6, -8)$, therefore T is parallel to $(6, 6, -8)$, therefore $r'(t)$ is parallel to $(6, 6, -8)$. This is the case if and only if $r'(t) \times (6, 6, -8) =$

$(0,0,0)$. We have

$$r'(t)\times(6,6,-8) = \begin{vmatrix} i & j & k \\ 3t^2 & 3 & 4t^3 \\ 6 & 6 & -8 \end{vmatrix} = (-24-24t^3)i+(24t^2+24t^3)j+(18t^2-18)k,$$

and this is equal to $(0,0,0)$ only when $t=-1$. This corresponds to the point $r(-1)=(-1,-3,1)$ on the curve.