Math 223

Week 12 Homework due Tuesday, November 16

Sections 13.4, 13.5

Section 13.4, problems 1, 9, 11, 13, 17, 21, 22, 23

Hints:

11. Parameterize the circle using $r(t) = (4\cos t, 4\sin t)$.

21. In part (a), parameterize C using $r(t) = ((1-t)x_1 + tx_2, (1-t)y_1 + ty_2)$, $0 \le t \le 1$. In part (b), Use Green's Theorem with $F(x, y) = (\frac{y^2}{2}, \frac{x^2}{2})$ to calculate the work down around the polygon and argue that the result is twice the area, using part (a) to compute the work done along each side of the polygon.

22: I'll derive the formula for $\overline{x}$ and you can use this as a model for how to derive $\overline{y}$. We know that

$$\overline{x} = \frac{\int\int_D x\ dA}{A}.$$

This suggests we should find P and Q such that $Q_x - P_y = x$. I'll choose $Q = \frac{x^2}{2}$ and $P = 0$. Then

$$\overline{x} = \frac{\int\int_D x\ dA}{A} = \frac{\int\int_D x\ dA}{A} = \frac{\int\int_D Q_x - P_y\ dA}{A} =$$

$$\frac{\int_C P\ dx + Q\ dy}{A} = \frac{\int_C \frac{x^2}{2}\ dy}{A} = \frac{\int_C x^2\ dy}{2A}.$$

Now derive the formula for $\overline{y}$, given that

$$\overline{y} = \frac{\int\int_D y\ dA}{A}.$$

Section 13.5, problems 1, 7, 9, 11, 13, 15, 17, 25, 29

Hints:

11, 13, 15: F will be conservative iff curl $F = 0$ when F is defined everywhere. See also Example 3, page 759.

17. When a vector field is the curl of another vector field, it's divergence will be equal to zero. See Theorem 11 and Example 5, page 761.

25. Let $F = (P, Q, R)$ and $G = (p, q, r)$, where P, Q, R, p, q, r are all functions of x, y, and z. Compute both sides of the equation you are trying to prove and check that they are equal.

29. See the definitions of the symbols in the instructions for problems 28–30 on the same page.