Math 223

Week 8 Homework due Tuesday, October 19

Sections 12.5, 12.6

Section 12.5, problems 9, 19, 25, 27, 33, 35, 39, 45

Hints:

General Hint: Let x_1, x_2, x_3 denote x, y, z in some order. Suppose the region of integration R is between the planes $x_1 = a$ and $x_1 = b$. Suppose further that, through a plane parallel to the x_2x_3 plane and slicing through a particular value of x_1, the resulting cross-section of R can be described as the type I region $\phi_1(x_1) \leq x_2 \leq \phi_2(x_1)$, $\psi_1(x_1, x_2) \leq x_3 \leq \psi_2(x_1, x_2)$. Then

$$\int\int\int_R f(x,y,z)\, dV = \int_a^b \int_{\phi_1(x_1)}^{\phi_2(x_1)} \int_{\psi_1(x_1,x_2)}^{\phi_2(x_1,x_2)} f(x,y,z)\, dx_3\, dx_2\, dx_1.$$

9. The solid region of integration is between $z = 0$ and the largest value of z over the planar region of the xy-plane between $y = \sqrt{x}$, $y = 0$, $x = 1$. So use $x_1 = z$, $x_2 = x$, $x_3 = y$.

19. The cross-sections of the solid region of integration are parallel to the xy-plane, which suggests that $x_1 = z$.

25. It seems that $x_1 = x$, $x_2 = z$, $x_3 = y$.

27. Draw the region of integration, then decide on two bounding planes to determine x_1, then characterize the cross sections as type I to determine x_2 and x_3.

33. Here $x_1 = y$, $x_2 = x$, and $x_3 = z$. Now draw the region of integration and mimic #27.

Section 12.6, problems 7, 13, 17, 21, 25, 27

Hints:

General Hint: With cylindrical coordinates we express the triple integral

$$\int_a^b \int_{\phi_1(x)}^{\phi_2(x)} \int_{F(x,y)}^{G(x,y)} f(x,y,z)\, dz\, dx\, dy$$

as a double integral

$$\int_a^b \int_{\phi_1(x)}^{\phi_2(x)} h(x,y)\ dx\ dy,$$

where

$$h(x,y) = \int_{F(x,y)}^{G(x,y)} f(x,y,z)\ dz.$$

If the region of integration in the double integral can be expressed in polar coordinates by $x = r\cos\theta$, $y = r\sin\theta$, $\theta_1 \leq \theta \leq \theta_2$, $f(\theta) \leq r \leq g(\theta)$, then we can have

$$\int\int\int_R f(x,y,z)\ dV = \int_{\theta_1}^{\theta_2} \int_{f(\theta)}^{g(\theta)} rh(r\cos\theta, r\sin\theta)\ dr\ d\theta =$$

$$\int_{\theta_1}^{\theta_2} \int_{f(\theta)}^{g(\theta)} \int_{F(r\cos\theta, r\sin\theta)}^{G(r\cos\theta, r\sin\theta)} rf(r\cos\theta, r\sin\theta, z)\ dz\ dr\ d\theta.$$