Math 223

Week 5 Homework due Tuesday, September 28

Sections 11.7, 11.8

Section 11.7, problems 7, 27, 29, 31, 37, 43

Hints:

31. The equation of the plane can be expressed as $z = x + y - 1$. The typical point on the plane is $(x, y, x+y-1)$. The distance from this point to $(2, 1, -1)$ is $\sqrt{(x-2)^2 + (y-1)^2 + (x+y-2)^2}$. You want to find the (x, y) pair that minimizes this expression. So the mathematical model is Minimize $f(x, y) = \sqrt{(x-2)^2 + (y-1)^2 + (x+y-2)^2}$. Note that the domain of f is all pairs (x, y) without restriction.

37. Let's say the box intersects the xy plane at vertices $(A, B, 0)$, $(A, -B, 0)$, $(-A, B, 0)$, and $(-A, -B, 0)$. Then it will intersect the ellipsoid at the corners of the box, one of which is $(A, B, \sqrt{\frac{36-9A^2-36B^2}{4}})$. Therefore the height of the box is $2\sqrt{\frac{36-9A^2-36B^2}{4}}$. Since the box has cross-sectional area of $4AB$, its volume will be $8AB\sqrt{\frac{36-9A^2-36B^2}{4}}$. Now maximize the function $f(A, B) = 8AB\sqrt{\frac{36-9A^2-36B^2}{4}}$. Note that you cannot use unrestricted values of A and B, so be careful to state the restrictions on A and B, thereby finding the domain of f.

43. If the box has length L, width W, and height H, then its volume is $32000 = LWH$. On the other hand, the amount of cardboard comes from the base of area LW and the four sides of total area $LH+LH+WH+WH$. So you want to maximize $LW + 2LH + 2WH$ subject to the restriction $LWH = 32000$. Since $H = \frac{32000}{LW}$, you are really trying to maximize $f(L, W) = LW + 2L\frac{32000}{LW} + 2W\frac{32000}{LW} = LW + \frac{64000}{W} + \frac{64000}{L}$. Specify the domain of this function before beginning.

Section 11.8, problems 5, 15, 17, 19, 25, 31, 39

Hints:

25. Maximize $\sqrt{(x-2)^2 + (y-1)^2 + (z+1)^2}$ subject to $x + y - z = 1$.

31. If the box has length L, width W, and height H, then it intersects the ellipsoid at $(\frac{L}{2}, \frac{W}{2}, \frac{H}{2})$. So you are trying to maximize LWH subject to $9(\frac{L}{2})^2 + 36(\frac{W}{2})^2 + 4(\frac{H}{2})^2 = 36$.

39. The distance from (x, y, z) to the origin is $\sqrt{x^2 + y^2 + z^2}$. You want to maximize and minimize this subject to the two simultaneous conditions $x + y + 2z - 2 = 0$ and $z - x^2 - y^2 = 0$.