Math 223

Week 4 Homework due Thursday, September 23

Sections 11.5, 11.6

Section 11.5, problems 1, 5, 17, 23, 29, 31, 33

Hints:

31. You are asked to find $\frac{dC}{dt}$ at time $t = 20$. While you are not given formulas for $T(t)$ and $D(t)$, you can approximate the value of $T'(20)$ and $D'(20)$ by estimating the slopes of their respective tangent lines at $t = 20$, and you can read off the values of $T(20)$ and $D(20)$ from the graph.

33 (a) $V = lwh$, $\frac{dV}{dt} = \frac{\partial V}{\partial l}\frac{dl}{dt} + \frac{\partial V}{\partial w}\frac{dw}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt} = whl' + lhw' + lwh'$. We are not told what time t_0 it is when $l(t_0) = 1$ and $w(t_0) = h(t_0) = 2$, but we are given that at this moment $l'(t_0) = 2$, $w'(t_0) = 2$, and $h'(t_0) = -3$. Therefore, plugging in the numbers, we can compute $\frac{dV}{dt}(t_0) = (2)(2)(2) + (1)(2)(2) + (1)(2)(-3) = 6$ m^3/sec.

Section 11.6, problems 7, 13, 17, 21, 26, 31, 37, 45

Hints:

7. Scale v down to a unit vector $u = \frac{v}{|v|}$ first.

13. The direction from P to Q is $Q - P = (3, -4)$.

21. In other words, for which values of (x, y) is $\nabla f(x, y)$ parallel to $(1, 1)$? Where $(f_x, f_y) = (\lambda, \lambda)$ for some λ.

23. The problem implies $T(x, y, z) = \frac{k}{\sqrt{x^2+y^2+z^2}}$ for some constant of proportionality k.

26. Parts (a) and (b) can be answered by considering partial derivatives at (60,40,966).

37. The gradient vector is perpendicular to the tangent line to the level curve. So the equation of the tangent line must be $\nabla f(2, 1)\cdot(x-2, y-1) = 0$. (Note that if (x, y) is a point on the tangent line then the vector $(x - 2, y - 1)$ is parallel to the line.)

45. Write the paraboloid in the form $f(x, y, z) = 0$. Write the ellipsoid in the form $g(x, y, z) = 0$. Let $r(t) = (x(t), y(t), z(t))$ be the curve of intersection

(it's not relevant what the actual formulas for $x(t)$, $y(t)$, and $z(t)$ are) to these level curves. Then $f(r(t)) = 0$ and $g(r(t)) = 0$, therefore the derivatives satisfy $\nabla f(r(t)) \cdot r'(t) = 0$ and $\nabla g(r(t)) \cdot r'(t) = 0$. You need to find the direction of $r'(t_0)$, at which point $r(t_0) = (-1, 1, 2)$, to determine the equation of the line tangent to the curve of intersection. In other words, solve the simultaneous equations $\nabla f(-1, 1, 2) \cdot r'(t_0) = 0$ and $\nabla g(-1, 1, 2) \cdot r'(t_0) = 0$. If you set $r'(t_0) = (a, b, c)$, you should be able to determine (a, b, c) to within a constant of proportionality.