Math 223

Week 1 Homework due Tuesday, August 31

Sections 10.6, 10.7, 10.8

Section 10.6, problems 1, 3, 9, 19, 21, 29, 31, 34

Hints:

31. This distance from (x, y, z) to $(-1, 0, 0)$ is $\sqrt{(x+1)^2 + y^2 + z^2}$. The distance from (x, y, z) to the plane $x = 1$ is $|x - 1|$. Equate these two distances, square them to eliminate the square root symbol and the absolute value symbol, then rearrange the equation and complete the square as necessary.

34. The curve of intersection is the set of points that satisfy both equations simultaneously. Rearrange the first equation to $x^2 + 2y^2 - z^2 + 3x - 1 = 0$. If (x, y, z) satisfies both equations, then it makes both equations equal to zero. So set these things equal to each other, then rearrange the resulting equation and check that it defines a plane (review Section 10.5 if necessary).

Section 10.7, problems 7, 15, 23, 35, 46, 49, 55, 58

Hints:

7. One way to decide the direction of increasing t is to look at the tangent vector at time $t = 0$.

23. Substitute the formulas for x, y, and z into the equation and verify that $z^2 = x^2 + y^2$.

35. Plot the tangent vector $r'(\frac{\pi}{4})$ as a vector displacement from the position $r(\frac{\pi}{4})$.

46. See Example 8, page 564. The unit tangent vector $T(t)$ has length one and points in the same direction as the tangent vector $r'(t)$.

55. To find the angle between the vectors $r_1'(0)$ and $r_2'(0)$, use the formula on page 532.

Section 10.8, problems 1, 7, 14, 15, 24, 31, 37, 41

Hints:

24. Use Formula 11 to find a formula for curvature as a function of x, then find the maximum and minimum value of $\kappa(x)$ (if such exist) using Calculus

I methods. This determines the critical values of x, which in turn determine the positions on the curve where max and min curvature exist.

37. The normal plane at a point on a curve is determined by the point $r(t)$ and the normal to the curve $N(t)$ at that point. To find the scalar equation of the normal plane, use Formula 7 on page 549 (consult Example 4 on that page also).

41. The normal plane is defined by the vector $N(t)$, which is perpendicular to the normal plane. The plane $6x + 6y - 8z = 1$ is perpendicular to the vector $(6, 6, -8)$ (review the section on planes, pp. 548–549). You will want to find the time at which $N(t)$ is parallel to the vector $(6, 6, -8)$. (Two vectors U and V are parallel if $U = \lambda V$ for some real number λ – see Figure 7, page 524. Alternatively, U and V are parallel if $U \times V = 0$ using the cross product.)