Math 223 Final Exam Topics

1. Parameterize a surface. See Section 13.6. This requires expressing x, y, and z in terms of two parameters u and v, then describing how u and v vary in terms of inequalities. A common parameterization of the surface $z = f(x, y)$ is $(x, y, f(x, y))$, as in Example 6, but you should also be able to do parameterizations based on polar coordinates as in Examples 5 and 6 and on spherical coordinates as in Example 4. For practice, take a second look at all the homework problems from 13.6 through 13.9.

2. Compute the surface integral of the scalar field $f : \mathbb{R}^3 \to \mathbb{R}$. See Section 13.7. This requires parameterizing the surface as $r(u, v) = (x(u, v), y(u, v), z(u, v))$, expressing the range of values of u and v as a system of inequalities D, computing the length of the cross product $r_u \times r_v$, then computing the double integral of $f(x(u, v), y(u, v), z(u, v))||r_u \times r_v||$ over D. For practice, take a second look at the homework problems in this section.

3. Compute the flux of a vector field $F : \mathbb{R}^3 \to \mathbb{R}^3$ across a surface S. This requires parameterizing the surface as above, then computing the double integral of the scalar field $F(x(u, v), y(u, v), z(u, v)) \cdot (r_u \times r_v)$ over the region D which describes how u and v vary.

4. Compute the work done by a vector field $F : \mathbb{R}^3 \to \mathbb{R}^3$ around a closed, positively oriented path C that lies on a surface S using Stoke's Theorem. The content of Stoke's theorem is that this work is equal to the flux of curl F across the region of S enclosed by C. You will need to parameterize the surface, including appropriate inequalities that define how u and v are varying, then compute the curl of F, then set up the integral which computes the flux of curl F across the region bounded by S and C. See Section 13.8 and review the homework problems in this section.

5. Compute the flux of a vector field $F : \mathbb{R}^3 \to \mathbb{R}^3$ across a closed orientable surface S using the Divergence Theorem (orientable just means there is a notion of inside or outside the surface, as opposed to the Mobius Strip which has only one side even though it's two-dimensional). The content of the Divergence Theorem is that this flux is equal to the triple integral of the divergence of F over the solid region bounded by S. You will need to compute the divergence of F and set up the triple integral describing the solid region. See Section 13.9 and review the homework problems in the section. Also, review the section on setting up triple integrals in rectangular coordinates (12.5), in cylindrical coordinates (12.6), and spherical coordinates (12.7).