The $r_e$ Transistor model

Remind

Q-poiint

$re = 26 \text{mv/IE}$
BJT AC Analysis

Three amplifier configurations,

Common Emitter

Common Collector (Emitter Follower)

Common Base
Process
Replace transistor with small-signal model.
Replace capacitors with short-circuits (at midband frequency caps have relatively low impedance).
Replace DC voltage sources with short-circuits. Replace DC current sources with open-circuits).
The simulation results include the following,

- $I_B = 6.172\, \mu A$
- $I_C = 0.9932\, mA$
- $I_E = 0.999\, mA$
- $V_C = 8.027\, V$
- $V_B = 0.6433\, mV$
Output voltage, $v_o$; i.e., collector voltage.

The peak voltages are $+142.692\text{mV}$ and $-147.4\text{mV}$, an average of about $145\text{mV}$.
BJT AC Analysis

\[ A_{\text{vb}} = \frac{v_{\text{cp}}}{v_{\text{bp}}} \] where "p" means the peak value.

\[ v_{\text{cp}} = -i_{\text{cp}} \left( \frac{R_c}{r_o} \right) \]
\[ = -\beta i_{\text{bp}} \left( \frac{R_c}{r_o} \right) = -\beta i_{\text{bp}} R_c \quad \text{when } r_o \text{ is } >> R_c \]

\[ v_{\text{bp}} = i_{\text{bp}} (1 + \beta) r_e \]

\[ A_{\text{vb}} = \frac{v_{\text{cp}}}{v_{\text{bp}}} \]
\[ \sim -\frac{R_c}{r_e} \quad \text{and} \quad = 26\text{mV}/0.999\text{mA} = 26 \text{ ohms} \]

\[ |A_{\text{vb}}| = \frac{4000}{26} = 153.8 \quad \text{which is close to the simulation value, 145 which is the result of 145mV/1mV}. \]
A_{ib} = \frac{i_{cp}}{i_{bp}} = \beta \text{ where “p” means the peak value. }

The AC collector current peak-to-peak is about $1.0302 \text{mA} - 0.957434 \text{mA} = 0.072766 \text{mA}$, so the peak is about 0.36383mA.

The AC base current is nearly identical to current passing through the capacitor, C1, and the peak is about 220nA.

Using these values the transistor beta is 165.
\[ Z_{ib} = \frac{v_{bp}}{i_{bp}} \]
\[ = \frac{i_{bp}(1 + \beta)r_c}{i_{bp}} \]
\[ = (1 + \beta)r_c \]

Using the beta information from the simulation, 
\[ Z_{ib} = 4300 \text{ ohms} \]

\[ Z_i = R_B // Z_{ib} \]

In this circuit since \( R \) is so large, \( Z_i \sim Z_{ib} \)

Output Impedance, \( Z_o \)

Turn off the input signal, \( V_s = 0 \).

The input current is zero so the collector current is zero.
\[ i_b = 0 \text{ and } i_c = 0 \]

Connect a test signal generator to the circuit output

\[ \begin{align*}
Z_o &= \frac{v_x}{i_x} \\
\quad &= \frac{v_x}{(R_c // r_o)} \\
\quad &= R_c // r_o
\end{align*} \]
The simulation results include the following,

- $I_B = 4.053 \mu A$
- $I_{R1} = \mu A$
- $I_C = 0.7565 mA$
- $I_E = 0.7614 mA$
- $V_C = 8.974 V$
- $V_B = 3.835 mV$
Output voltage, $v_o$, i.e., collector voltage.

The peak voltages are $+16.721\text{mV}$ and $-16.753\text{mV}$, an average of about $16.4\text{mV}$.
BJT AC Analysis

\[ A_{vb} = \frac{v_{cp}}{v_{bp}} \text{ where ”p” means the peak value.} \]

\[ v_{cp} = -i_{cp}(Rc/r_o) = -\beta i_{bp} (Rc/r_o) = -\beta i_{bp} Rc \quad \text{when } r_o \text{ is } >> Rc \]

\[ v_{bp} = i_{bp} (1 + \beta)(R_{E1} + r_e) \]

\[ A_{vb} = \frac{v_{cp}}{v_{bp}} \]

\[ \sim -\frac{Rc}{(R_{E1} + r_e)} \quad \text{and} \quad = 26mV/0.7614mA = 31.15 \text{ ohms} \]

\[ |A_{vb}| = \frac{4000}{(200 + 31.15)} = 17.31 \text{ which nearly identical to the simulation value, which is the result of 17.3.} \]

\[ Rit = \frac{v_b}{i_b} = (1 + \beta_{ac})(r_e + R_{E1}) = 121(26.597 + 200) = 27.418k \]

\[ A_{vb} = \frac{v_c}{v_b} \]

\[ v_e = i_b(1 + \beta_{ac})R_{E1} \]

\[ v_b = i_b(1 + \beta_{ac})(r_e + R_{E1}) \]

\[ v_c = -\beta_{ac}(i_b)R_c \]

\[ A_{vb} = \frac{v_c}{v_b} \quad = \frac{-\beta_{ac}(i_b)R_c}{i_b(1 + \beta_{ac})(r_e + R_{E1})} \quad = \frac{-\beta_{ac}(R_c)}{\beta_{ac}(r_e + R_{E1})} . \]
BJT AC Analysis

\[
(1 + \beta ac)(re + Re1) = \frac{-120 \times 3.9k}{121 \times 226.597} = -17.069
\]

Notice that the gain is stabilized by including the Re1 resistor. Without Re1 the gain is very dependent on the value of the transistor gain, \( \beta \), which varies quite a bit and results in variations in base current which in turn changes the value of \( re \).
\[ \text{Rot} = \frac{v_x}{i_x} = R_c = 3.9k \]

\[ v_x - i_x (R_c) = 0 \]
\[ RL' = \frac{R_c}{RL} = \frac{R_c \cdot RL}{R_c + RL} = 795.92 \]

\[ R_{OL} = RL' \]
With RL attached the voltage gain is reduced,

\[ A_{vb} = \frac{v_c}{v_b} = -\beta_{ac} R_L' \cdot \frac{1 + \beta_{ac}(r_e + R_e)}{1 + \beta_{ac}(r_e + R_e)} \]

which is a low level of gain, 

\[ A_{vb} = 3.482 \]
$A_{v_{s}}$.

$V_{s}' = \frac{V_{s} \cdot R_{th}}{R_{s} + R_{th}}$.
BJT AC Analysis

\[ Rs' = \frac{R_s R_{th}}{R_s + R_{th}} \]

\[ V_s' = i_b[R_s' + (1 + \beta_ac)(r_e + R_{e1})] \]

\[ \frac{V_s R_{th}}{R_s + R_{th}} = i_b[R_s' + (1 + \beta_ac)(r_e + R_{e1})] \]

\[ V_c = i_b R_L' = V_s \frac{R_{th}}{R_s + R_{th}} \times \frac{R_L}{R_s' + (1 + \beta_ac)(r_e + R_{e1})} \]
Another way to tackle the gain calculation simplifies the math,

\[
\frac{V_b}{V_s} = \frac{\frac{R_{th}}{R_{it}}}{\frac{R_{th}}{R_{it}} + R_s} = \frac{5889}{5889 + 50} = 0.99158
\]

From earlier analysis,

\[
A_{vb} = \frac{V_c}{V_b} = -\frac{\beta_{ac} R'L'}{(1 + \beta_{ac})(r_e + R_e1)}
\]

Combining the above two equations,

\[
(V_b/V_s) \times (V_c/V_b) = V_c/V_s
\]

so that,

\[
\frac{V_c}{V_s} = \frac{\frac{R_{th}}{R_{it}}}{\frac{R_{th}}{R_{it}} + R_s} \times \frac{-\beta_{ac} R'L'}{(1 + \beta_{ac})(r_e + R_e1)} = 0.99158 \times 3.482 = 3.4528
\]

High-frequency analysis will use this approach of breaking down the problem into pieces followed by multiplication of the individual gains.
To summarize,
1. Compute $R_{it}$
2. Compute the gain $V_s/V_b$ using the value of $R_{it}$.
3. Compute the gain $V_c/V_b$.
4. Multiply the gains to get the overall gain $V_c/V_s$. 
Inverting nature of amplifier -
  Increase input voltage,
  increases base and collector current,
  decreases collector voltage because of more drop across load resistance.
The emitter follower circuit acts as a buffer between stages.
The input and output voltage waveforms are nearly identical; almost independent of the value of the load resistor.
$\beta_{ac} = 120$  assume  $r_o$ is very large  $re2 = 66.93$
Without the load resistor,

\[
v_b - i_b(1 + \beta_{ac})r_e + r_a = 0
\]

\[
R_{it} = \frac{v_b}{i_b} = (1 + \beta_{ac})(r_e + r_a) = 121(66.39 + 40000) = 4.848M
\]
With the load resistor,

\[ \beta_{ib} = \frac{v_b}{i_b} = (1 + \beta_{ac})(r_e + \frac{R_a}{R_L}) = 121(66.39 + 493.9) = 67.8k \]

\[ \frac{v_e}{v_b} = \frac{(1 + \beta_{ac})\frac{R_a}{R_L}}{(1 + \beta_{ac})(r_e + \frac{R_a}{R_L})} = \frac{\frac{R_a}{R_L}}{r_e + \frac{R_a}{R_L}} \]

Rit = \frac{v_b}{i_b} = (1 + \beta_{ac})(r_e + \frac{R_a}{R_L}) = 121(66.39 + 493.9) = 67.8k
Output Resistance

\[
\ Ro = \frac{v_x}{i_x}
\]

Sum currents entering the emitter node.

\[
\begin{align*}
i_x + \beta_{ac}i_b + i_b - \frac{v_x}{(R_a//R_L)} &= 0 \\
i_x + (1 + \beta_{ac})i_b - \frac{v_x}{(R_a//R_L)} &= 0 \\
i_b &= -\frac{v_x}{[R_c + (1 + \beta_{ac})r_e]} \\
i_x + (1 + \beta_{ac})(-\frac{v_x}{[R_c + (1 + \beta_{ac})r_e]}) - \frac{v_x}{(R_a//R_L)} &= 0
\end{align*}
\]
BJT AC Analysis

\[ ix - vx \left[ \frac{1}{\frac{Rc}{1 + \beta ac} + re2} + \frac{1}{Ra/RL} \right] = 0 \]

Two resistors in parallel, \((\frac{Rc}{1 + \beta ac} + re2)\) and \(Ra/RL\) equals \(Ro\).
BJT AC Analysis
Common-Base Configuration

(a) Common-Base Configuration

(b) Common-Base Configuration

\[ I_c = \alpha I_e \]

\[ \text{re} := 26 \text{ mV} \frac{\text{mV}}{\text{IE}} \]

\[ Z_o := \frac{V_o}{I_o} \]

\[ Z_i := \frac{V_i}{I_i} \]

\[ Z_i := \text{re} \]

\[ Z_o := \infty \]
Example

IE = 4 mA
α = 0.98
RL = 560 ohms
Vi = 2 mV

Compute re, Ii, Vo, Av, Ai
Common-Base Configuration
BJT AC Analysis

\[ Z_i = \frac{R_E}{r_e} \]

\[ Z_o = R_c \]

\[ A_v = \frac{V_o}{V_i} \]

\[ V_o = I_C R_C \]

\[ V_i = I_E r_e \]

\[ A_v = \frac{\alpha I_E R_C}{I_E r_e} = \frac{\alpha R_C}{r_e} \]

\[ A_i = \alpha \]
Current Source Circuit
Current Mirror Circuit