Math 345-60 Abstract Algebra I
Questions for Section 4: Groups

1. A group is a binary structure \( \langle G, * \rangle \) satisfies three properties. Describe these properties.

2. Why is \( \langle \mathbb{Z}, + \rangle \) a group?

3. Why is \( \langle \mathbb{Z}, - \rangle \) not a group?

4. Why is \( \langle \mathbb{Q}, \cdot \rangle \) a group?

5. Why is \( \langle \mathbb{Z}, \cdot \rangle \) not a group?

6. What property should * have to say that \( \langle G, * \rangle \) is an abelian group?

6. Is the group \( \langle LL_2(\mathbb{R}), \cdot \rangle \) (the group of matrices with non-zero determinant) abelian? Why or why not?

7. What is the inverse of 5 in the group \( \langle \mathbb{Q}, \cdot \rangle \)?

8. What is the inverse of 5 in the group \( \langle \mathbb{Z}, + \rangle \)?

9. Let \( \langle G, * \rangle \) be a group. Prove that \( a * b = a * c \) implies \( b = c \) for all \( a, b, c \in G \) (compare with the proof of Theorem 4.15, page 41).

10. Let \( \langle G, * \rangle \) be a group. Prove that \( (a * b * c)' = c' * b' * a' \) using Corollary 4.18.

Homework for Section 4, due ?? (only the starred problems will be graded):
3, 7, 10*, 12*, 14*, 20*, 33, 35*

Hints:
10. Just do the problem for \( n = 3 \).

12. Just do the problem for \( 2 \times 2 \) diagonal matrices \( \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \).

14. Just do the problem for \( 2 \times 2 \) diagonal matrices.

20. Fill out the rest of Table 4.22 to make a group. There are two possibilities. For part (c), use \( n = 2 \).

35. Use the cancelation laws.