Math 316-01 Intermediate Analysis

Questions for Section 7: Functions

1. What is a function?
2. What is the domain of the function \( f : A \to B \)?
3. What is the codomain of the function \( f : A \to B \)?
4. What is the range of the function \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = x^2 \)?
5. Is the function \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = x^2 \) injective?
6. Is the function \( f : \mathbb{R} \to [0, \infty) \) defined by \( f(x) = x^2 \) surjective?
7. Is the function \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = 2x + 1 \) bijective?
8. Let \( f : \mathbb{R} \to \mathbb{R} \) be defined by \( f(x) = 2x + 1 \). What is the image of the set \([0, 1)\)?
9. Let \( f : \mathbb{R} \to \mathbb{R} \) be defined by \( f(x) = 2x + 1 \). What is the preimage of the set \([0, \infty)\)?
10. What is the inverse of the function \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = 2x + 1 \)?
11. Is the function \( f : (-\infty, 0] \to [0, \infty) \) defined by \( f(x) = x^2 \) bijective? Prove your answer.
12. What is the formula for the inverse of the function in Question 11?
13. What is the identity function?
14. Let \( f : A \to B \) be a bijective function. What are the domain and codomain of the inverse function?
Homework for Section 7, due ??? (only the starred problems will be graded):

1, 2, 3*, 6, 7*(b, d, g), 9*(c, e, f), 10*, 13, 14*, 17*, 28*

Hints:

10(a). Try constructing a function by cases:

\[
f(x) = \begin{cases} 
\text{something} & \text{if } x \text{ is even} \\
\text{something else} & \text{if } x \text{ is odd}
\end{cases}
\]

17. Part (a) is true – give a proof in the usual way. Part (b) is false, so you will have to construct an example where \( f(S) \subseteq f(T) \) and \( S \not\subseteq T \). My suggestion: Pick \( S \) and \( T \) first so that \( S \not\subseteq T \), then define \( f \) so that \( f(S) \subseteq f(T) \).

28. Look for an example where the sets \( A, B, \) and \( C \) are finite sets. Since \( g \circ f \) is bijective, we must have \( |A| = |C| \) (set cardinalities (sizes) are equal). Use this as your starting point.