1. Define what it means for $f$ to have a limit $L$ at $c$, being careful to specify the domain and codomain of $f$. Express the definitions in terms of quantifiers and logical connectives. Need $f(c)$ be defined?

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x$. We wish to show that $\lim_{x \to c} f(x) = 2c$. Let $\epsilon > 0$ be given. We must choose $\delta > 0$ small enough so that $0 < |x - c| < \delta$ guarantees $|f(x) - 2c| < \epsilon$. How small must $\delta$ be?

3. Study Example 20.6 carefully. One way to show that $\lim_{x \to c} f(x) = L$ is to show that $|f(x) - L| = |S(x)||x - c|$, where $S(x)$ is a quantity that is bounded by some $M \geq 0$ when $|x - c|$ is small enough. So we need to choose $\delta$ so that $0 < |x - c| < \delta$ ensures both that $|S(x)| \leq M$ and $|x - c| < \frac{\epsilon}{M}$. Given this, prove that if $f(x) = x^2 + x$ then $\lim_{x \to 2} f(x) = 6$.

4. In Theorem 20.8, page 193, it is proved that if $f : D \to \mathbb{R}$ and if $\lim_{x \to c} f(x) = L$, then every sequence $(s_n) \subseteq D \backslash \{c\}$ converging to $c$ satisfies $\lim_{n \to \infty} f(s_n) = L$. Method: let $\epsilon > 0$ be given. Then there exists $\delta > 0$ such that $0 < |x - c| < \delta$ guarantees $|f(x) - L| < \epsilon$. So we just need to choose $N$ so that $n > N$ guarantees that $|s_n - c| < \delta$. This guarantees $n > N$ implies $|f(s_n) - L| < \epsilon$. To prove that the statement of the theorem is if and only if, it is necessary to show that if $\lim_{x \to c} f(x) \neq L$, there is a sequence $(s_n)$ in $D \backslash \{c\}$ which converges to $c$ such that $(f(s_n))$ does not converge to $L$. How do you express in logical terms that $f(x)$ does not converge to $L$ as $x \to c$? (Negate the logical statement in Question 1). What role does $\frac{1}{n}$ play in this negation (3rd sentence of second paragraph of the proof of Theorem 20.8).

5. Read Example 20.16, page 195, very carefully. It is proved that $\lim_{x \to 1} f(x) = 1$, where

$$f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x - 1} & x \neq 1 \\ 5 & x = 1. \end{cases}$$

Since $f(x)$ is defined for all $x \in \mathbb{R}$, it is clear that the domain of $f$ is $D = \mathbb{R}$. Given this, how is Theorem 20.13 being applied? What justifies the statement $\lim_{x \to 1} f(x) = \lim_{x \to 1}(2x - 1) = 2(1) - 1 = 1$?
Homework for Section 20, due ??? (only the starred problems will be graded):
1, 2, 3∗(a, b, h), 4∗, 6∗(b), 7∗(b), 9∗(b, c), 15∗, 16∗

Hints:
4. Use the method of Question 3 and Example 20.6.
6(b). Give the $\epsilon$-$\delta$ proof.
7(b). Use the logic of Question 5.
9(b). Use the method of Example 20.11.
9(c). Why is $|x \sin(\frac{1}{x})| \leq x$?
15. $(f \circ g)(x)$ is short for $f(x)g(x)$. $U$ is short for $N(c, u) = (c - u, c + u)$ for some $u > 0$. Use the method of Question 3.