Math 316 Intermediate Analysis
Exam 4 Preparation

1. Section 33: Prove that \( \sum a_n \) converges or diverges, where each \( a_n \geq 0 \). Method: compare \( \sum a_n \) to \( \sum b_n \) using the Comparison Test. You will need to justify your comparison by proving that \( a_n \leq b_n \) or \( a_n \geq b_n \geq 0 \) for all \( n \). Then prove that \( \sum b_n \) converges or diverges using either the ratio test, root test, or integral test. See Exercises 33.3 and 33.5.

2. Section 34: Find the radius of convergence \( R \) and the interval of convergence \( C \) of \( \sum a_n x^n \). Method: radius of convergence can be determined looking at \( \sum |a_n x^n| \) and applying the root or ratio test. Interval of convergence requires testing \( x = R \) and \( x = -R \), assuming \( R \neq +\infty \) and \( R \neq 0 \). When using the alternating series test, make sure all hypotheses of that test are satisfied. See Exercises 34.3 and 34.5.

3. Section 35: Prove or disprove that a sequence of functions \( (f_n) \) converges uniformly to a function \( f \) on a set \( S \) using the definition for all \( \epsilon > 0 \) there exists \( N \) such that \( n > N \) implies \( |f_n(x) - f(x)| < \epsilon \) for all \( x \in S \). See Exercises 35.4, 35.16.

4. Section 35: Prove that \( \sum f_n \) converges uniformly to a function \( f \) on \( S \) using the Weierstrauss M-test (Theorem 35.11, page 324). See Exercise 35.19.

5. Section 36: Integrate a power series term-by-term using Corollary 36.5. See Problem 36.11 and Example 36.6. In order to use Corollary 36.5, you must first verify the hypotheses of that Corollary. You can do that using the Weierstrauss M-Test.