1. Let \( \epsilon > 0 \) be given. We must find \( \delta \) such that \( 0 < |x - 0| < \delta \) implies 
\[
\left| \frac{f(x) - f(0)}{x} - 0 \right| < \epsilon.
\]

But 
\[
\left| \frac{f(x) - f(0)}{x} - 0 \right| \text{ is equal to either } |x \sin x| \leq |x| < \delta \text{ or } |x^2 \cos x| \leq |x|^2 < \delta^2,
\]
so we will choose \( \delta = \min(\epsilon, \sqrt{\epsilon}) \).

2. Define \( f : [a, b] \to \mathbb{R} \) by \( g(x) = f(x) - x^2 \). Since \( f \) and \( x^2 \) are differentiable on \([a, b]\), so is \( g \). That makes \( g \) continuous on \([a, b]\). Hence \( g \) is continuous on \([a, b]\), differentiable on \((a, b)\), and satisfies \( g(a) = f(a) - a^2 = 0 \) and \( g(b) = f(b) - b^2 = 0 \). By Rolle’s Theorem, there exists \( c \in (a, b) \) such that \( g'(c) = 0 \). Since \( g'(x) = f'(x) - 2x \), we have \( 0 = g'(c) = f'(c) - 2c = 0 \), which implies \( f'(c) = 2c \).

3. The derivatives of \( f(x) \) are 
\[
f(x), f'(x), f''(x), f'''(x), f^{'''}(x) = \ln(1-x), -(1-x)^{-1}, -(1-x)^{-2}, -2(1-x)^{-3}, -6(1-x)^{-4}.
\]
These are all continuous on \([0, \frac{1}{2}]\). Therefore, using \( x_0 = 0 \), we have 
\[
p_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 = -x - \frac{x^2}{2} - \frac{x^3}{3}
\]
and 
\[
|R_3(x)| = \left| -6(1-c_x)^{-4}x^4 \right| < \left| 6\left(\frac{1}{2}\right)^{-4}\frac{x^4}{24} \right| = \left| \frac{6}{24} \right| = \frac{1}{4}
\]
for some \( c_x \in (0, \frac{1}{2}) \). Using \( x = \frac{1}{2} \), we have \( \ln(\frac{1}{2}) = f(\frac{1}{2}) \approx p_3(\frac{1}{2}) = -\frac{1}{2} - \frac{1}{8} - \frac{1}{24} \) with error \( < \frac{1}{4} \). Note that \( -\frac{2}{3} = -0.666666 \cdots \) whereas \( \ln(\frac{1}{2}) = -0.693147 \cdots \).

4. Let \( P = \{0, 5 - \delta, 5 + \delta, 10\} \). Then \( U(f, P) = 500(5 - \delta) + 1000(2\delta) + 1000(5 + \delta) \) and \( L(f, P) = 500(5 - \delta) + 500(2\delta) + 1000(5 + \delta) \), therefore \( U(f, P) - L(f, P) = 500(2\delta) = 1000\delta \). Choosing \( \delta = \frac{\epsilon}{1001} \), we obtain \( U(f, P) - L(f, P) < \epsilon \). So \( P = \{0, 5 - \frac{\epsilon}{1001}, 5 + \frac{\epsilon}{1001}, 10\} \).

5. Let \( c \in (0, \pi) \) and define \( f : [c, \pi] \to \mathbb{R} \) by \( f(x) = \frac{\sin x}{x} \). Given that \( f'(x) = \frac{x \cos x - \sin x}{x^2} \) for all \( x \in [c, \pi] \), and given that \( f' \) is continuous on \([c, \pi]\), we know that \( f' \) is integrable and by the Fundamental Theorem of Calculus we have 
\[
\int_c^\pi \frac{x \cos x - \sin x}{x^2} \, dx = \int_c^\pi f' \, dx = f(\pi) - f(c) = \frac{\sin \pi}{\pi} - \frac{\sin c}{c} = -\frac{\sin c}{c}.
\]
Therefore

\[ \int_0^\pi \frac{x \cos x - \sin x}{x^2} \, dx = \lim_{c \to 0^+} -\frac{\sin c}{c} = -\lim_{c \to 0^+} \frac{\cos c}{1} = -1. \]