Math 316 Intermediate Analysis

Exam 3 Preparation

1. Section 25: Prove that a function does or does not have a derivative at a point. See Exercises 3, 8, 11. Be prepared to use the definition \( f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \) and, if necessary, the \( \epsilon-\delta \) definition of limit.

2. Section 26: Use the Mean Value Theorem (including its special case, Rolle’s Theorem) to prove that a function has a given property. See Exercises 10, 18. You should be able to demonstrate that the function meets both hypotheses of the Mean Value Theorem (or all three hypotheses of Rolle’s Theorem).

3. Section 28: Use Taylor’s Theorem (28.2) to approximate a function value by a Taylor polynomial and estimate the error in the calculation using the formula for the remainder \( R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x - x_0)^{n+1} \) without using a calculator. Make sure you verify that all the hypotheses of Taylor’s Theorem are met. See Exercises 4, 7

4. Section 29: Prove that a given function is integrable using Theorem 29.9. That is, given \( \epsilon > 0 \), find \( P \vdash [a, b] \) such that \( U(f, P) - L(f, P) < \epsilon \). See Exercises 16, 17.

5. Section 31: Compute an improper integral. This requires applying Definition 31.8 or Definition 31.10 and Theorem 31.5 (taking care to demonstrate that the definitions and the hypotheses of the theorem are met). See Exercise 17.