Math 375
Week 9 Homework due Thursday, March 25

Section 5.3: problems 3, 8, 11, 13, 21, 26, 31
Hints:
8. Assume sandwiches of the same type are indistinguishable. In part (a), the sequences are

person 1 sandwich, person 2 sandwich, ..., person 9 sandwich.

In part (b), just keep track of how many of each kind of sandwich.

26. Assume the rings are all different and have names $r_1, r_2, \ldots, r_9$. Part (a) can be modeled as all sequences $(X_1, X_2, X_3, X_4)$ where $X_1 \cup X_2 \cup X_3 \cup X_4 = \{r_1, r_2, \ldots, r_9\}$ and no two of these set have a ring in common. Empty sets are allowed. In part (b), you must keep track of not only the different partitions $(X_1, X_2, X_3, X_4)$ but also the order in which the rings appear on each finger.

Section 5.4: problems 1, 21, 37, 47, 49, 61, 62
Hints:
61. The typical arrangement with $k$ runs of 0s can be identified with the sequence of numbers

$$(x_0, y_1, x_1, y_2, x_2, \ldots, y_k, x_k),$$

where the arrangement begins with $x_0 \geq 0$ ones, followed by $y_1 \geq 1$ zeros, followed by $x_1 \geq 1$ ones, followed by $y_2 \geq 1$ zeros, ..., followed by $y_k \geq 1$ ones, followed by $x_k \geq 0$ ones. You can choose the sequences $(x_0, x_1, \ldots, x_k)$ and $(y_1, y_2, \ldots, y_k)$ independently of each other. Since there are $n$ 0s and $m$ 1s, we must have

$$x_0 + x_1 + \cdots + x_k = m$$

and

$$y_1 + y_2 + \cdots + y_k = n.$$  

62. You can model a random arrangement by two sequences: a binary sequence of 13 0s and 39 1s with $k$ runs of zeros (0 represents heart, 1 represents non-heart), and a sequence which describes which card is represented by each character in the string. Use the multiplication principle: first choose the binary string, then multiply by the number of sequences of cards that correspond to this binary string.