Math 122
Week 7 Homework due Thursday, Mar 4
Sections 8.1, 8.2

Section 8.1: problems 9, 11, 13, 15, 17, 21, 33, 35, 37

9. Factor \( n^2 \) in numerator and denominator to simplify.

11. Factor as \( \frac{1}{3}(\frac{2}{3})^n \).

13. Cancel out common factors, given that \((n + 2)! = 1 \times 2 \times \cdots \times n \times (n + 1) \times (n + 2)\) and \( n! = 1 \times 2 \times \cdots \times n \).

14. Factor \( n \) out of the numerator and \( n^2 \) out the denominator.

17. Factor \( e^n \) out of the numerator and \( e^{2n} = (e^n)^2 \) out of the denominator.

21. Note that \( |\cos^2 n| \leq 1 \) for all \( n \), therefore \( -1 \leq \frac{1}{\sqrt{n}} \leq 1 \).

37. First note that \( a_1 = \sqrt{2} \), \( a_2 = \sqrt{2a_1} \), \( a_3 = \sqrt{2a_2} \), .... These values are all less than 2: the first one is. Suppose the \( n^{th} \) one is. Then the \( n + 1^{th} \) one is \( \sqrt{2a_n} \leq \sqrt{2} \cdot \sqrt{2} = 2 \). So the sequence is bounded using mathematical induction. It is also increasing because \( \frac{a_{n+1}}{a_n} = \frac{\sqrt{2a_n}}{\sqrt{2a_n}} = \sqrt{\frac{2}{\sqrt{2}}} \geq \frac{\sqrt{2}}{\sqrt{2}} = 1 \).

So there must be a limit \( L \) by the monotone sequence theorem. Note that \( L = \lim a_n = \lim \sqrt{2a_{n-1}} = \sqrt{2} \lim a_{n-1} = \sqrt{2} \cdot \sqrt{L} \). Now solve for \( L \).

Section 8.2: problems 3, 5, 9, 11, 13, 19, 21, 27, 29

Hints:

3. Factor this as \( 5(1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \cdots) \).

5. This can be expressed equivalently as \( 5 \sum_{n=0}^{\infty} (\frac{2}{3})^n \).

9. Factor this as \( \frac{1}{3}(1 + \frac{1}{2} + \frac{1}{3} + \cdots) \).

11. Use the Test For Divergence, formula 7 on page 425.

13. Rewrite this as \( \sum (\frac{1}{3})^n + (\frac{2}{3})^n \) and use Theorem 8, part (ii), bottom of page 425.

19. Expand \( \frac{2}{(n-1)(n+1)} \) by partial fraction decomposition, as in Example 6, then compute the first few partial sums to see what they are.

21. Same hint as in #19.

27. Use Theorem 4, page 422, with \( a = \frac{1}{3} \) and \( r = \frac{1}{3} \).
29. This can be expressed in the equivalent form $1 + \sum_{n=1}^{\infty} \left( \frac{\cos x}{2} \right)^n$. Use Theorem 4, page 422, with $a = \frac{\cos x}{2}$ and $r = \frac{\cos x}{2}$. 