Section 7.5, problems 5, 11, 13, 23, 41

Hints:

5. Similar to problem 6, which we worked out in class.

11. Similar to problem 10, worked out in class. Dissect this problem as follows: Lift the entire cable and coal $\Delta x$ feet and compute the work done. Now cut off $\Delta x$ feet of cable at the top and lift the shorter cable and coal $\Delta x$ feet. Now cut another $\Delta x$ feet of cable at the top and lift the even shorter cable and coal $\Delta x$ feet. Express this as a Riemann sum of expressions of the form pounds $\times$ feet, the compute the integral to get an answer in units of foot-pounds.

13. Similar to problem 12, worked out in class. Dissect this problem similarly to problem 11. Lift bucket and rope $\Delta x$ meters, using original weight of bucket and original weight of the rope. Now cut off $\Delta x$ meters of rope and lift the lighter bucket (mass at the end of the first lift) and the shorter rope another $\Delta x$ meters. Keep on going until fully lifted. Recall that weight is mass $\times$ acceleration, where acceleration is 9.8 meters per second per second. The main problem is to figure out how much the bucket and the rope weigh at the beginning of each lift.

23. Similar to problem 24, worked out in class. Dissect the semicircular region into horizontal rectangles. Estimate the force on the each rectangle using $F = PA$, where $P$ is the pressure at the appropriate depth and $A$ is the area of the rectangle. If you make a coordinate system with $y = 0$ corresponding to the top of the figure and increasing in the downward direction, and if $x = 0$ corresponds to the midpoint of the top of the figure, then the equation of the semicircle satisfies $x^2 + y^2 = 10^2$.

41. Draw the figure first so you can predict approximately where the centroid will be (as a check on your work).
Section 7.6, problems 1, 9, 15, 37, 39, 45

Hints:

37. Set this up as \( \frac{dP}{P(1000-P)} = 0.00008 \, dt \). Integrate both sides, using partial fraction decomposition for the left-hand side. You should get a constant of integration. Use \( P(0) = 100 \) to determine the constant of integration. Now solve for \( P \) to get a formula in terms of \( t \). To answer the last question, solve for \( t \) in \( P(t) = 900 \).

39. (a) Let \( t \) represent hours since 8:00 AM. Let \( y \) represent the fraction of people who have heard the rumor at the time represented by \( t \). We are told that the rate of change of \( y \) (in other words, \( \frac{dy}{dt} \)) is proportional to the product of \( y \) and \( 1-y \). Use \( k \) as the constant of proportionality. (b) Make sure your solution includes a constant of integration \( C \). (c) The initial conditions can be interpreted as \( y(0) = \frac{80}{1000} = .08 \), \( y(4) = \frac{1}{2} = .50 \). Use these to find both \( k \) and \( C \). Now find \( t_0 \) such that \( y(t_0) = .90 \), and interpret the answer in terms of hours since 8:00 AM.

45. Similar to problem 46, worked out in class. Let \( y \) represent gallons of alcohol in tank at time \( t \), where \( t \) measures number of minutes of mixing. Model \( \frac{dy}{dt} \) as rate of alcohol pumped into tank (gallons per minute) minus rate of alcohol in pumped out of tank (gallons per minute). Note that \( A \) gallons of low-alcohol beer contains \( .04A \) gallons of alcohol, etc. The initial condition can be interpreted as \( y(0) = .04(500) = 20 \). The percentage of beer after an hour is \( \frac{y(60)}{\text{gallons of beer in tank after an hour}} \times 100 \).