Math 122 Spring 2010
Selected Solutions to Homework 5 Problems

Section 7.4

7. Same as length of \( y = \frac{1}{3} \sqrt{x}(x - 3), 1 \leq x \leq 9. \)

25. Note that if \((a, b)\) lies on the curve \( x^{2/3} + y^{2/3} = 1 \), then so do \((-a, b)\), \((a, -b)\), and \((-a, -b)\). So this curve is symmetric about both the \(x\)-axis and the \(y\)-axis. We will compute the length of the curve in the first quadrant, then multiply by 4. Note that the \(x\) and \(y\)-intercepts are \((\pm1, 0)\) and \((0, \pm1)\), so in the first quadrant the curve satisfies \(0 \leq x \leq 1. \)

We have \( x^{2/3} + y^{2/3} = 1. \) Therefore

\[
y = (1 - x^{2/3})^{3/2},
\]

\[
y' = \frac{3}{2}(1 - x^{2/3})^{1/2} \cdot -\frac{2}{3} x^{-1/3} = -(x^{-1/3})(1 - x^{2/3})^{1/2},
\]

\[
1 + (y')^2 = 1 + x^{-2/3}(1 - x^{2/3}) = x^{-2/3},
\]

\[
4 \int_0^1 x^{-2/3} \, dx = 4 \lim_{a \to 0^+} \int_a^1 x^{-2/3} \, dx = 4 \lim_{a \to 0^+} 3x^{1/3} \bigg|_a^1 = 12 \lim_{a \to 0^+} (1 - a^{1/3}) = 12.
\]

29. Solution sketched out in class.