Math 122 Spring 2010
Selected Solutions to Homework 4 Problems

Section 7.1

31. The parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ intersect at $(-c, 0)$ and $(c, 0)$, assuming $c > 0$. The area between them is

$$\int_{-c}^{c} (c^2 - x^2) - (x^2 - c^2) \, dx = 2 \int_{-c}^{c} c^2 - x^2 \, dx = \frac{8c^3}{3}. $$

Setting this equal to 576 we obtain $c = 6$.

Section 7.2

25. We can obtain a right-circular cone by revolving a straight line from $(0, 0)$ to $(h, r)$ about the $x$-axis. The equation of this line is $y = \frac{r}{h} x$. So the volume is

$$\int_{0}^{h} \pi r^2 \frac{x^2}{h^2} \, dx = \frac{\pi r^2 h}{3}. $$

35. The base of the figure is all points in the $xy$ plane which satisfy $x^2 \leq y \leq 1$. In other words, above the parabola $y = x^2$ and below the line $y = 1$. When the solid is cut by a line perpendicular to the $y$ axis, the resulting cross-section is a square. To estimate the volume of the solid, dissect the volume by horizontal lines through $y_0 < y_1 < \cdots < y_n$, where $y_0 = 0$ and $y_1 = 1$. To estimate the volume of the slice between $y_{i-1}$ and $y_i$, use the area of the cross section through $y_i^* \,(\text{between } y_{i-1} \text{ and } y_i)$ and multiply by $\Delta_i$, the distance between $y_{i-1}$ and $y_i$. Note that since $y = x^2$ on the parabola, the horizontal line through $y = y_i^* \,$ intersects the parabola at $x = -\sqrt{y_i^*}$ and $x = +\sqrt{y_i^*}$. The distance between these two points is $2\sqrt{y_i^*}$, therefore the area of the cross-section is $4y_i^*$. So the volume of the slice is approximately $4y_i^* \Delta_i$. The total volume is approximately $\sum_{i=1}^{n} 4y_i^* \Delta_i$, and the exact volume is $\int_{0}^{1} 4y \, dy = 2$. 