19. If \( f(x) = \cos x \) then \( f^{(n)}(x) \) is equal to either \( \sin x \), \( -\sin x \), \( \cos x \), or \( -\cos x \). In all cases \( |f^{(n)}(0)| \leq 1 \). So \( M_n = 1 \) can be used for all \( n \). The error term for the \( n^{th} \) Taylor polynomial expanded about \( x = 0 \) is no greater than
\[
\frac{M_{n+1}}{(n+1)!}|x-0|^{n+1} = \frac{|x|^{n+1}}{(n+1)!}.
\]
We proved in class that these expressions approach zero as \( n \to \infty \), no matter what \( x \) is. Therefore the Taylor polynomial for \( \cos x \) converges to \( \cos x \) for all \( x \).

55.
\[
e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots
\]
\[
e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \cdots = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots
\]
\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots
\]
Therefore
\[
e^{-x^2} \cos x = \left(1 - \frac{x^2}{2!} + \frac{x^4}{3!} + \cdots\right) \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right) = 1 - \frac{3x^2}{2} + \frac{25x^4}{24} + \cdots.
\]