Choose two of the following six:

(1) Prove ONE of the remaining cases of the Saarkovskii Theorem. The cases are:
   (a) Prove that if \( f \) has period \( p \cdot 2^m \) with \( p \) odd, then \( f \) has period \( q \cdot 2^m \) for any odd \( q \) with \( q > p \).
   (b) Prove that if \( f \) has period \( p \cdot 2^m \) with \( p \) odd, then \( f \) has period \( 2^l \) for all \( l \leq m \).
   (c) Prove that if \( f \) has period \( p \cdot 2^m \) with \( p \) odd, then \( f \) has period \( q \cdot 2^m \) for any even \( q \).

(2) Given \( f : I \to I \), we construct a new map, \( D(f) \), called the double of \( f \) whose periodic points will have exactly twice the period of those of \( f \) (and adds an additional fixed point). To construct this map, divide \( I \) into thirds and compress the graph of \( f \) into the upper left corner of \( I \times I \) as in the figure. Lastly, linearly extend the map as depicted in the figure.

(3) Let \( j \) be an integer. Construct a map of period \( 2^j \), but not of period \( 2^l \) for all \( l > j \). Does your map have points of period \( 2^m \) for \( m < j \)?

(4) Construct a map of period 10 and all periods implied by period 10 in the Saarkovskii ordering, but none others.

(5) Does there exist a map with periods of order \( \{1, 2, 2^2, ..., 2^j, ...\} \) but no other periods? (Hint: Can you take the limit of \( f_j \) as \( j \to \infty \), where \( f_j \) is the map you might have constructed in Exercise 3.)

(6) Let \( A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \). Prove that the \( Tr(A^k) = Tr(A^{k-2}) + Tr(A^{k-1}) \). What does this imply about \( \Sigma_A \)?